1. At room temp $\tau = 3ps$

HW#3 EE 482 Solutions 1/10

$$\frac{1}{\tau} = \frac{1}{\tau_{lattice}} + \frac{1}{\tau_{impurities}}$$

(Both are equally likely)

$$\tau_{lattice} = \tau_{imp} = 2\tau = 6ps$$

a) Now temp is changed so that:

prob lattice scattering is <u>halved</u>; so lifetime is doubled $\tau_{lattice} = 12ps$

prob. ionized impurity is $1.5\times$: $\tau_{impurity} = \frac{6ps}{1.5} = 4ps$

$$\frac{1}{\tau} = \frac{1}{\tau_{lat}} + \frac{1}{\tau_{imp}} = \frac{1}{12ps} + \frac{1}{4ps}$$

$\tau_p = 3ps$, no change

At lower temp, lattice vibration is reduced $\therefore$ $\tau_{lattice}$ increases. Also at lower temp carriers move slower so the probability of ionized impurity scattering increases.

Therefore the temp was decreased.

b) $m_e^* = .5m_o$; $p = 10^{17}\,cm^{-3}$ (typo in HW)

$$V_d = -\frac{q\,\tau_p}{m_e^*}\mathcal{E}$$

At <u>room temp</u> $V_d = -\frac{(1.6\times10^{-19}c)(3\times10^{-12}s)}{.5(9.11\times10^{-31}kg)}\left(1000\frac{V}{cm}\right)$

$$V_d = -1.05\times10^{3}\,\frac{CVs}{kg\,cm} = 1.05\times10^{3}\,\frac{\cancel{kg}m^2\,\cancel{s}}{\cancel{s^2}\,\cancel{kg}\,cm} = -1.05\times10^{7}\,\frac{cm}{s}$$

Ignoring minority carriers $J = -neV_d$ ← (J_n)

$$J_p = -(10^{17}cm^{-3})(1.6\times10^{-19}c)(-1.05\times10^{7}\,\tfrac{cm}{s}) = 1.69\times10^{4}\,\frac{A}{cm^2}$$

1c) Max current density

$$J_{max} = n_e V_{d\,max}$$

where $V_{d\,max} = 6\times10^{6}$

$$J_{max} = (10^{17}\,cm^{-3})(1.6\times10^{-19}\,C)(6\times10^{6})$$

$$J_{max} = 9.6\times10^{4}\ \frac{A}{cm^2}$$

The system (b) would actually be limited by $v_{d\,max}$

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2. IC Resistor w/ diffused Arsenic into Si

@ $N_a = 2\times10^{17}\,cm^{-3}$ of Boron

$\overline{Nd} = \frac{N'}{x_j} = \frac{6\times10^{12}\,cm^{-2}}{.1\,\mu m} = 6\times10^{17}\,cm^{-3}$

average $n \hat{=} N_d - N_a = 4\times10^{17}\,cm^{-3}$

$p = \frac{n_i^2}{n} = \frac{(1.5\times10^{10})^2}{4\times10^{17}} = 562\ cm^{-3}$

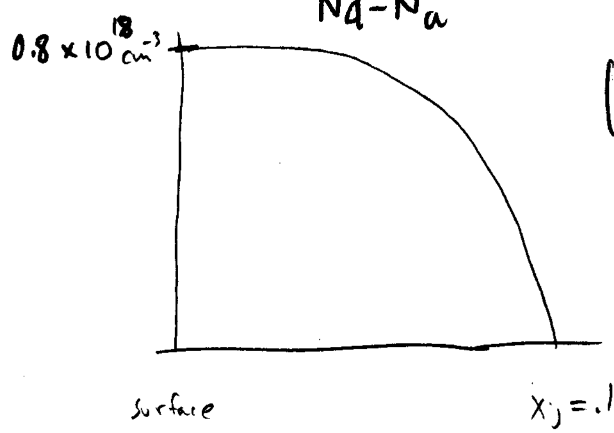
(a) In bulk

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)} \cong \frac{1}{q\mu_p p}$$

$\mu_p \cong 180\,\frac{cm^2}{V\cdot s}$

$\underline{\underline{\rho = 0.17\ \Omega\text{-cm}}}$

Nd−Na



Mobility based on total doping.

Use $N_a + \overline{N_d} = 8\times10^{17}\,cm^{-3}$

$\bar{\mu}_n = 300\,cm^2/Vs$

$\bar{\mu}_p = 100\,cm^2/Vs$

b) $$R_\square = \frac{1}{q\int_0^{x_j}\mu_p p\,dx} \cong \frac{1}{q\bar{\mu}_p\int_0^{x_j}p\,dx} = \frac{1}{q(300\,\frac{cm^2}{V\cdot s})(6\times10^{12}\,cm^{-2})} = 3.5\times10^{3}\,\Omega$$

$$\underline{\underline{= 3.5\,k\Omega}}$$

c) $$R = \frac{L}{W}R_\square = \frac{2\,\mu m}{0.5\,\mu m}3.5\,k\Omega = 14\,k\Omega$$

3. GaAs (direct bandgap) $N_a = 10^{16}\,cm^{-3}$

@ Room temp 300K

$P_o = 10^{16}\,cm^{-3}$ $\quad n = \dfrac{n_i^2}{P} = \dfrac{(1.8\times10^6)^2}{10^{16}} = 3\times10^{-4}\,cm^{-3}$

a) Assuming low level injection, at steady-state

$$\underbrace{\Delta n = \Delta p}_{} = \frac{G_L}{k(p+\cancel{n}^{\approx 0})} = \frac{10^{20}\,cm^{-3}\,\cancel{s^{-1}}}{10^{-8}\,cm^{+3}\,\cancel{s^{-1}}\,(10^{16}\,cm^{-3})} = 10^{12}\,cm^{-3}$$

LLI is ok

where G_L is the generation rate of $\underbrace{e^- \text{-hole pairs}}_{}$.
and $k = 10^{-8}\,cm^3/s$

$$\%\text{ change in } p = \frac{10^{12}}{10^{16}}\times 100 = .01\%$$

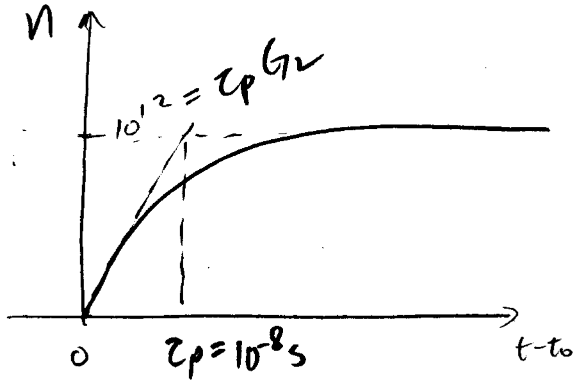
$$\%\text{ change in } n = \frac{10^{12}}{3\times10^{-4}} = 3.33\times10^{17}\%\,!$$

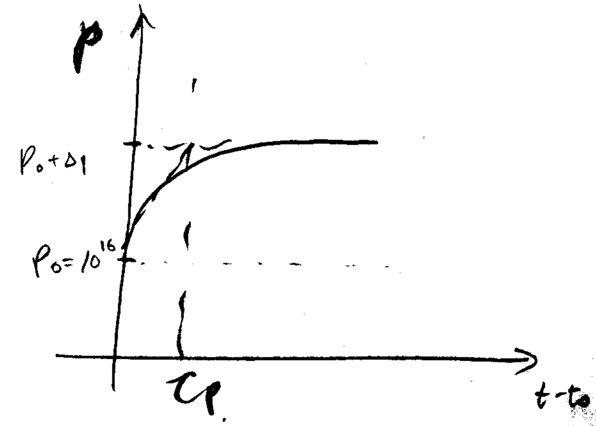
b) Light on at $t = t_o$ $\quad \Delta p(t_o) = 0 \quad \dfrac{dp}{dt} = G_L - \dfrac{\Delta p}{\tau_p}$

$$\Delta p(t) = A + Be^{-(t-t_o)/\tau_p} \qquad \Delta p(\infty) = 10^{12}\,cm^{-3}$$

$$\Delta p(t) = \Delta p\left(1 - e^{-\frac{(t-t_o)}{\tau_p}}\right); \quad \tau_p = \frac{1}{k(n_o+p_o)} = \frac{1}{10^{-8}\,10^{16}} = 10^{-8}\,s$$

$$\Delta p(t) = 10^{12}\left(1 - e^{-\left(\frac{t-t_o}{10^{-8}s}\right)}\right)$$





3b @ 50% at what time

$$.5\,\Delta p = .5(10^{12}) = 10^{12}\left(1 - e^{-\frac{t-t_0}{10^{-8}}}\right)$$

$$(.5) = 1 - e^{-\frac{t-t_0}{10^{-8}}}$$

$$e^{-\frac{t-t_0}{10^{-8}}} = 1 - .5 = .5$$

$$-\frac{t-t_0}{10^{-8}} = \ln(.5)$$

$$-(t-t_0) = -.69\times10^{-8}$$

$$\boxed{t - t_0 = .69\times10^{+8}\ s}$$

c) $G_L \cong 10^{26}$ no longer LLI

$$\Delta n = \Delta p = \frac{G_L}{k(n_0+\Delta p)} \qquad \Delta p^2 + \Delta p\, n_0 - \frac{G_L}{k} = 0$$

$$(\Delta p)^2 + 10^{16}\Delta p - 10^{34} = 0$$

$$\frac{\Delta p}{p_0} = \frac{9.51\times10^{16}}{10^{16}}\cdot 100 = 951\% \uparrow \qquad \Delta p = \frac{-10^{16} + \sqrt{(10^{16})^2 - 4(-10^{34})}}{2(1)}$$

$$\frac{\Delta n}{n_0} = \frac{9.51\times10^{16}}{3.24\times10^{-4}} = 2.9\times10^{22}\% \uparrow \qquad \Delta p = 9.51\times10^{16}\,cm^{-3}$$

+ No irradiation : $N_d + N_a \cong N_a = 10^{16}$; $\mu_n = 7000$; $\mu_p = 200$

$$\rho = (q[\mu_n n + \mu_p p])^{-1} = (1.6\times10^{-19}(200\times10^{16}))^{-1} = \boxed{3.125\ \Omega\cdot cm}$$

+ Irradiation

$p = 10^{16} + 9.51\times10^{16} = 1.05\times10^{17}\,cm^{-3}$

$n \cong \Delta p = 9.51\times10^{16}\,cm^{-3}$ with $N_t - 2\times10^{17}$ $\mu_p = 200$ $\mu_n = 3500$

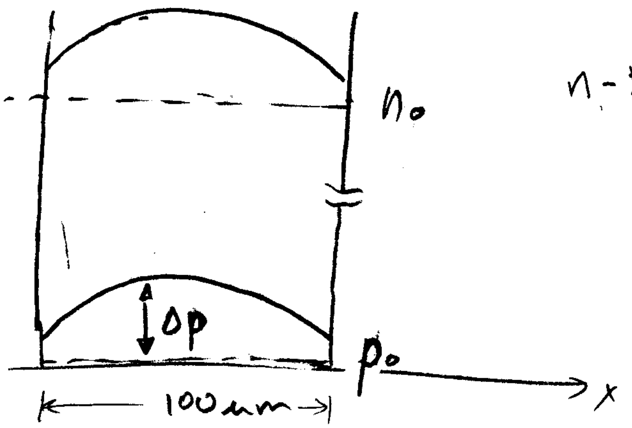
$$\rho = [1.6\times10^{-19}(200\cdot1.05\times10^{17} + 3500\cdot 9.51\times10^{16})]^{-1} \doteq \boxed{.018\ \Omega\cdot cm}$$

4. n-type Si $\quad N_d = 10^{17}\ cm^{-3}$

$G_L = 10^{18}\ cm^{-3}s^{-1}$

The top & bottom Recomb. velocity $= 10^5 \frac{cm}{s} = S$

Assume $\tau_n = 10\mu s$, $\tau_p = 20\mu s$ $\quad T = 300K$



n-type
solve for Δp because it is easier then $\Delta n = \Delta p$ since pairs are generated together

Assuming low level injection and bulk generation

$$\frac{\partial \Delta p}{\partial t} \xrightarrow{} 0\ \text{s-s} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L \quad @ \text{ Steady State } \frac{\partial \Delta p}{\partial t} = 0$$

Diffusion approximation, so ignore minority carrier drift

$$\frac{\partial^2 \Delta p}{\partial x^2} = \frac{\Delta p}{D_p \tau_p} - \frac{G_L}{D_p}$$

Assume Soln:

$$\Delta p = A_1 e^{-x/L_n} + A_2 e^{x/L_n} + G_L \tau_p$$

where $\quad L_p = \sqrt{D_p \tau_p} = \sqrt{(6.5\ \tfrac{cm^2}{s})(20\mu s)} = 11.4 \times 10^{-3} cm = 114\mu m$

because $D_p = \frac{kT}{q}\mu_p = \frac{1.38\times 10^{-23}\frac{J}{K}(300K)}{1.6\times 10^{-19}\ C} \cdot 250 = 6.5\ \frac{cm^2}{s}$

using $\mu_p = 250\ \frac{cm^2}{V \cdot s}$ from plot

$L_p = 114 \times 10^{-4}\ cm = .0114\ cm$

Boundry Conditions:

$$\frac{d\Delta p}{dx} = A\cdot a\cdot e^{ax} - B\cdot a\, e^{-ax}$$

$$D_p \left.\frac{d\Delta p}{dx}\right|_{x=0} = D_p a (A-B) \overset{!}{=} s\cdot\left(G_L\tau_p + A + B\right) \qquad (1)$$

$$D_p \left.\frac{d\Delta p}{dx}\right|_{x=L} = D_p a\left(Ae^{aL} - Be^{-aL}\right) \overset{!}{=} s\left(G_L\tau_p + Ae^{aL} + Be^{-aL}\right) \qquad (2)$$

⇒ two equations with two unkowns (A and B)

$$(1) \Rightarrow \quad A\left(D_p a - s\right) + B\left(-D_p a - s\right) = s\,G_L\tau_p$$

$$\Rightarrow B = \frac{A\left(D_p a - s\right) - s\,G_L\tau_p}{D_p a + s} \qquad (1)^*$$

plug into (2) (first rewrite)

$$A\left(D_p a\, e^{aL} - s\, e^{aL}\right) + B\left(-D_p a\, e^{-aL} - s\, e^{-aL}\right) = s G_L\tau_p \qquad \Big|\, e^{aL}$$

$$A e^{2aL}\left(D_p a - s\right) - B\left(D_p a + s\right) = s G_L\tau_p\, e^{aL} \qquad \text{plug in } (1)^*$$

$$A e^{2aL}\left(D_p a - s\right) - \frac{A\left(D_p a - s\right) - s G_L\tau_p}{\cancel{D_p a + s}}\cancel{\left(D_p a + s\right)} = s G_L\tau_p\, e^{aL}$$

$$A\left(D_p a - s\right)\left(e^{2aL} - 1\right) = s G_L\tau_p\left(1 + e^{aL}\right)$$

$$A = \frac{s G_L\tau_p}{D_p a - s}\,\frac{e^{aL} + 1}{e^{2aL} - 1}$$

$$B = \frac{s G_L\tau_p}{D_p a + s}\,\frac{e^{aL} + 1}{e^{2aL} - 1} - \frac{s G_L\tau_p}{D_p a + s}$$

Now you just need to plug in the numbers (see attached plot)

$a = \sqrt{\frac{1}{\tau_p D_p}} \equiv \frac{1}{L_p}$ $\qquad L_p = 0.161\ cm = 16.1\,\mu m$

$D_p \cdot a = \sqrt{\frac{D_p}{\tau_p}} = 80.62\ \frac{cm}{s}$

$$A = \frac{10^5 \cancel{cm} \cdot 10^{18} cm^{-3} \cdot 20\mu \cancel{s}}{\cancel{s}\ \cancel{s}\ \underbrace{(80.62 \frac{cm}{s} - 10^5 \frac{cm}{s})}_{\approx -10^5 cm/s}} \frac{e^{L/L_p} + 1}{e^{2aL} - 1}$$

$$A = -2 \cdot 10^{15} cm^{-3} \cdot 2 \cdot 10^{-3} = -4 \cdot 10^{12} cm^{-3}$$

$$B \approx 2 \cdot 10^{15} cm^{-3} \cdot 2 \cdot 10^{-3} - 2 \cdot 10^{15} cm^{-3} = -1.99 \cdot 10^{15} cm^{-3}$$

$$G_L \tau_p = 10^{18} cm^{-3}/s \cdot 20\mu s = 2 \cdot 10^{15} cm^{-3}$$

$$\Delta p(x) = 2 \cdot 10^{15} cm^{-3} - 4 \cdot 10^{12} cm^{-3} e^{x/L_p} - 1.99 \cdot 10^{15} e^{-x/L_p}$$ $\qquad L_p = 16.1\mu m$

$\Rightarrow$ see attached plot $\qquad$ (remember $\Delta n = \Delta p$)

b) $p(0) = p_0 + \Delta p(0)$

$$\Delta p(0) = 2 \cdot 10^{15} cm^{-3} - 4 \cdot 10^{12} cm^{-3} - 1.99 \cdot 10^{15} cm^{-3} \approx 6 \cdot 10^{12} cm^{-3}$$

$$p(0) = 10^3 cm^{-3} + 6 \cdot 10^{12} cm^{-3} \approx 6 \cdot 10^{12} cm^{-3}$$ $\leftarrow$ very low because S is greater than $\frac{D_p}{L_n}$

$$n(0) = n_0 + \Delta n(0) = 10^{17} cm^{-3} + 6 \cdot 10^{12} cm^{-3} \approx 10^{17} cm^{-3}$$

4.

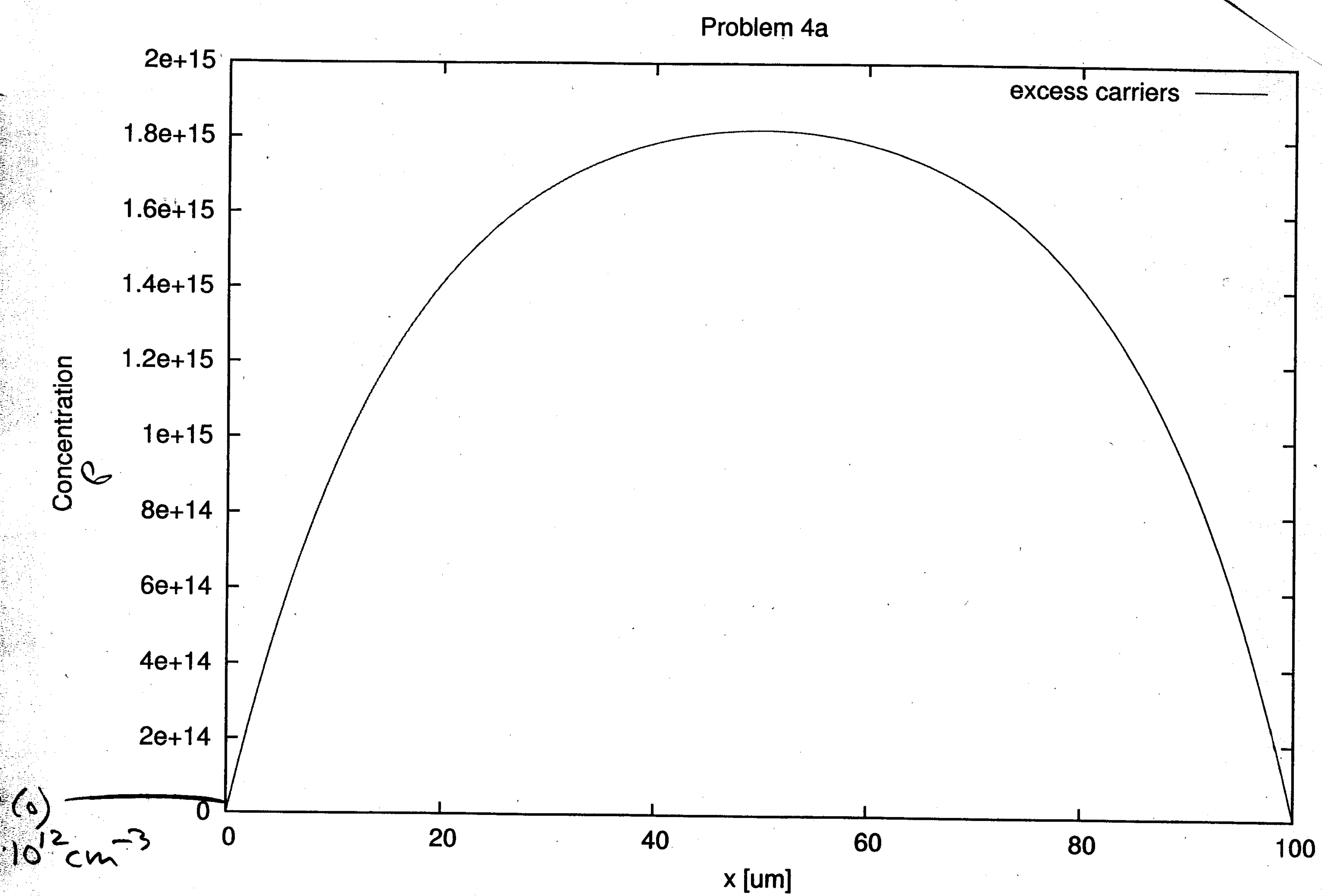
b)

$$\left(\frac{\text{light generated carriers}}{\text{surface recomb. carriers}}\right)^{-1} = \left(\frac{G_L}{S\cdot\Delta p(0)}\right)^{-1} = \left(\frac{10^{18}\,\cancel{cm^{-3}}\,\cancel{s^{-1}}}{10^{5}\,cm\,\cancel{s^{-1}}\cdot 6\cdot 10^{12}\,\cancel{cm^{-3}}}\right)^{-1} = 1.67^{-1}$$

$= 0.598 \approx 60\%$

just dividing the rates is good enough

the result is the same for e^- since you create h^+ e^- always in pairs.


Problem 4a
excess carriers
Concentration
p
2e+15
1.8e+15
1.6e+15
1.4e+15
1.2e+15
1e+15
8e+14
6e+14
4e+14
2e+14
0
(0)
10^{12} cm^{-3}
0
20
40
60
80
100
x [um]

5. $\rho = 1\ \Omega\text{-cm}$ p-type   Si $10^{12}\,cm^{-3}$ G-R

$$\rho = \frac{1}{e\mu_p N_a} = 1\ \Omega\cdot cm$$

From plot
$N_a = 1.3\times10^{16}\,cm^{-3}$
$p \simeq N_a = 1.3\times10^{16}\,cm^{-3}$

$$\tau_n = \frac{1}{V_{thn}\sigma_n N_t} = \frac{1}{(10^7 s^{-1})(10^{-15}cm^2)(10^{12}cm^{-3})}$$
$\tau_n = 1\times10^{-4}\,s$

use $\text{Recomb} = U = \dfrac{np - n_i^2}{\tau_n(p+n_i) + \tau_p(n+n_i)}$

where $\tau_p = \dfrac{1}{V_{thp}\sigma_p N_t} = \dfrac{1}{(6\times10^6)(10^{-15}cm^2)(10^{12}cm^{-3})} = 1.67\times10^{-4}\,s$

Assuming low-level injection (check later)

$G_L = \dfrac{\Delta n}{\tau_n} \longrightarrow \Delta p = G_L\tau_n = (10^{18}\,cm\,s)(1.0\times10^{-4}\,s)$
$\Delta p = 1\times10^{14}$

At S-S $U = G_L$

$P = P_o + \Delta p = (1.3\times10^{16} + 1\times10^{14}\,cm^{-3}) \cong 1.3\times10^{16}\,cm^{-3}$

$n = \dfrac{n_i^2}{P_o} + \Delta p = \dfrac{(1.5\times10^{10})^2}{1.3\times10^{16}} + 1\times10^{14} \cong 1\times10^{14}\,cm^{-3}$

since $\Delta p \simeq \Delta n$ Check low-level assumption $\Delta p \ll N_a$ ✓

b) $G_L = 10^{24}\,cm^{-2}s^{-1} \Rightarrow$ High level injection

$$R_i = \frac{(\cancel{p_o} + \Delta p)(\cancel{n_o} + \Delta n) - \cancel{n_o p_o}}{\tau_n(\cancel{p_o} + \Delta p + \cancel{n_i}) + \tau_p(\cancel{n_o} + \Delta n + \cancel{n_i})}$$
; using P_o & n_i can be neglected in comparison to Δp terms
- $\Delta n \triangleq \Delta p$

$$\frac{\Delta p^2}{\Delta p(\tau_n + \tau_p)} = G_L \Rightarrow \Delta p = G_L(\tau_n + \tau_p)$$

$\tau_n = .0001$

$\Delta p = 10^{24}(.000267)$
$\Delta p = 2.67\times10^{20}$

5c) Calculate Gen Rate if minority carrier $n \ll n_o$

$$R = \frac{np - n_i^2}{\tau_n(p+n_i) + \tau_p(n+n_i)}$$

$$= \frac{-n_i^2}{\tau_n(p+n_i) + \tau_p(n_i)} \cong \frac{-n_i^2}{\tau_n p_o}$$

$\therefore$ Generation Rate $= -R = \dfrac{+(1.5 \times 10^{10})^2 (cm^{-3})(cm^{-3})}{(1 \times 10^{-4} s)(1.3 \times 10^{16})(cm^{-3})}$

$$= \underline{\underline{1.73 \times 10^{8}\ cm^{-3}\ s^{-1}}}$$