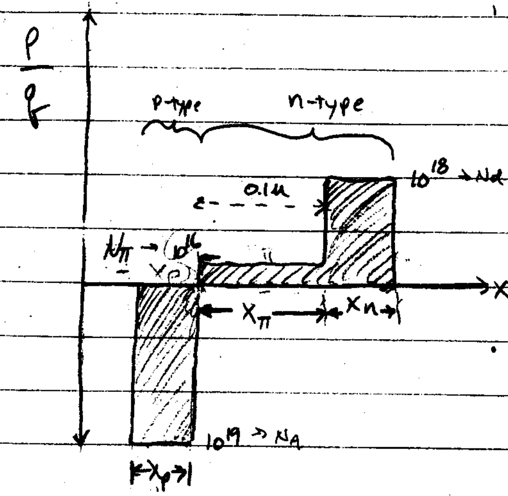
EE 482 HW #4 Soln's

1. (a)

SILICON: $\varepsilon_0 k = 11.7(8.85\times10^{-14})$



$N_A \sim 10^{19}\,cm^{-3}$ $N_\pi \sim 10^{16}\,cm^{-3}$ $N_D \sim 10^{18}\,cm^{-3}$

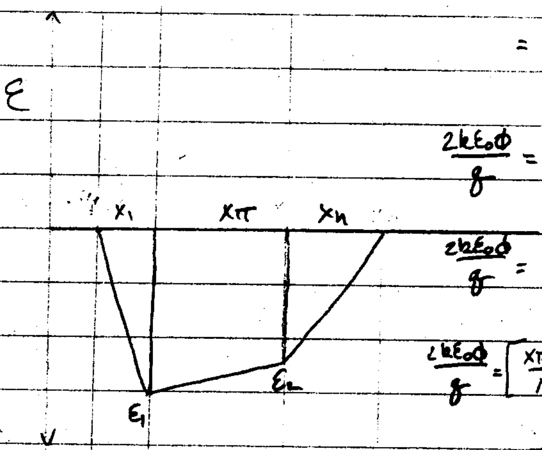
$$\boxed{\text{Charge Neutrality } x_p N_A = x_\pi N_\pi + x_n N_D}$$

$$\varepsilon_1 = -\frac{q N_a x_p}{k\varepsilon_0} \qquad \varepsilon_2 = -\frac{q N_d x_n}{k\varepsilon_0} = -\varepsilon_1 + \frac{q N_\pi x_\pi}{k\varepsilon_0}$$

$$\phi_i = \frac{kT}{q}\ln\left(\frac{N_d N_a}{n_i^2}\right) = 0.026\ln\left(\frac{10^{19}\,10^{18}}{(1.5\times10^{10})^2}\right) = 0.9967\,V$$

$$\phi_i = \frac{x_p(-\varepsilon_1)}{2} + \frac{x_\pi(-\varepsilon_2-\varepsilon_1)}{2} + \frac{x_n(-\varepsilon_2)}{2}$$

$$= \frac{x_p^2 q N_a}{2k\varepsilon_0} + \frac{x_\pi x_n q N_d}{2k\varepsilon_0} + \frac{x_\pi x_1 q N_a}{2k\varepsilon_0} + \frac{x_n^2 q N_d}{2k\varepsilon_0}$$



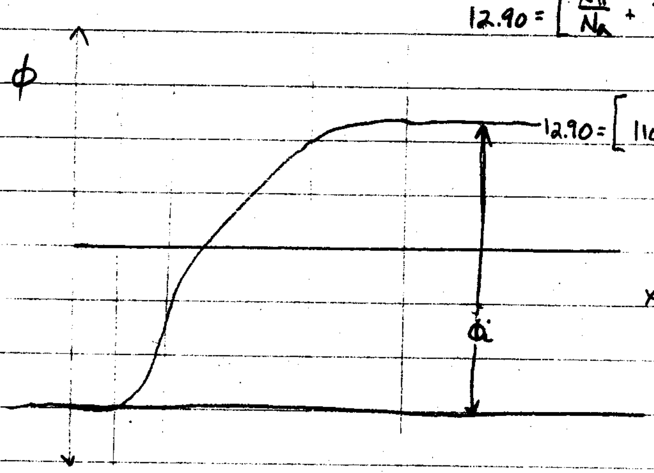
$$\frac{2k\varepsilon_0\phi}{q} = \left[x_p^2 N_a + x_\pi x_n N_d + x_\pi x_1 N_a + x_n^2 N_d\right]$$

$$\frac{2k\varepsilon_0\phi}{q} = \left[\frac{(x_\pi N_\pi + x_n N_D)^2}{N_a} + x_\pi x_n N_d + x_\pi(x_\pi N_\pi + x_n N_D) + x_n^2 N_d\right]$$

$$\frac{2k\varepsilon_0\phi}{q} = \left[\frac{x_\pi^2 N_\pi^2}{N_a} + \frac{2x_\pi N_\pi x_n N_D}{N_a} + \frac{x_n^2 N_D^2}{N_a} + x_\pi x_n N_D + x_\pi^2 N_\pi + x_\pi x_n N_D + x_n^2 N_D\right]$$

$$\frac{2k\varepsilon_0\phi}{q N_\pi x_\pi^2} = \left[\frac{x_\pi^2 N_\pi^2}{N_a N_\pi x_\pi^2} + \frac{2x_\pi N_\pi x_n N_D}{N_a N_\pi x_\pi^2} + \frac{x_n^2 N_D^2}{N_a N_\pi x_\pi^2} + \frac{x_\pi x_n N_D}{N_\pi x_\pi^2} + \frac{x_\pi^2 N_\pi}{x_\pi^2 N_\pi} + \frac{x_\pi x_n N_D}{x_\pi^2 N_\pi} + \frac{x_n^2 N_D}{x_\pi^2 N_\pi}\right]$$

$$12.90 = \left[\frac{N_\pi}{N_a} + \frac{2N_D}{N_a}\left(\frac{x_n}{x_\pi}\right) + \frac{N_D^2}{N_a N_\pi}\left(\frac{x_n}{x_\pi}\right)^2 + \frac{N_D}{N_\pi}\left(\frac{x_n}{x_\pi}\right) + 1 + \frac{N_D}{N_\pi}\left(\frac{x_n}{x_\pi}\right) + \frac{N_D}{N_\pi}\left(\frac{x_n}{x_\pi}\right)^2\right]$$



$$12.90 = \left[110\left(\frac{x_n}{x_\pi}\right)^2 + 200.2\left(\frac{x_n}{x_\pi}\right) + 1.001\right] \qquad \frac{x_n}{x_\pi} = 0.05761$$

$$\underline{\underline{x_n = 5.761\times10^{-7}\,cm}}$$

$$x_p = \frac{(0.1\times10^{-4})(10^{16}) + (5.761\times10^{-7})(10^{18})}{10^{19}} = \underline{\underline{6.761\times10^{-8}\,cm}}$$

1B) $\mathcal{E}_{max}$ OF PIN DIODE:

$$\mathcal{E}_1 = -\frac{q N_a x_p}{K\epsilon_0} = -\frac{(1.6\times10^{-19})(10^{19})(6.761\times10^{-8})}{(11.7)(8.85\times10^{-14})} = -1.095\times10^5 \ \frac{V}{cm}$$

$$\mathcal{E}_2 = -\frac{q N_d x_n}{K\epsilon_0} = -\frac{(1.6\times10^{-19})(10^{18})(5.761\times10^{-7})}{(11.7)(8.85\times10^{-14})} = -8.90\times10^4 \ \frac{V}{cm}$$

$\mathcal{E}_{max}$ OF PN DIODE WITHOUT INTRINSIC REGION

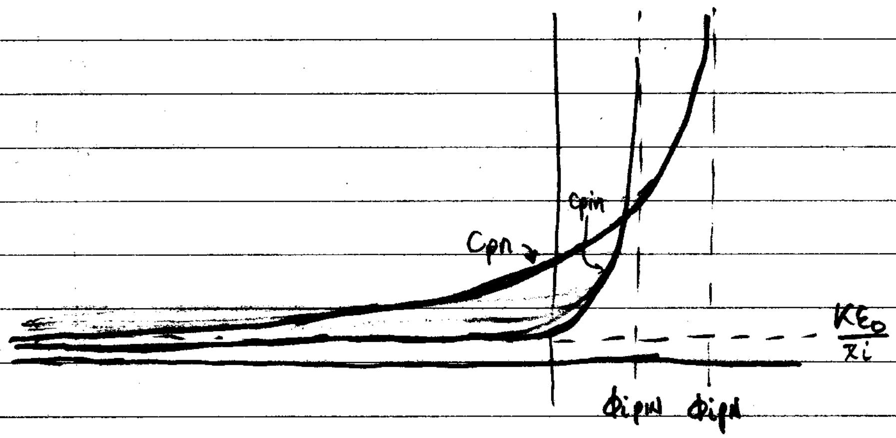
$$\mathcal{E}_{max} = -\left[\frac{2q\phi_i}{K\epsilon_0}\frac{N_a N_d}{(N_a+N_d)}\right]^{\frac{1}{2}} = -\left[\frac{2(1.6\times10^{-19})(0.9967)}{(11.7)(8.85\times10^{-14})}\frac{(10^{19})(10^{18})}{(10^{19}+10^{18})}\right]^{\frac{1}{2}} = -5.292\times10^5 \ \frac{V}{cm}$$

$\mathcal{E}_{max}$ WITHOUT INTRINSIC REGION IS ABOUT 5 TIMES LARGER BECAUSE THE INTRINSIC REGION INCREASES THE ~~AMOUNT~~ OF VOLTAGE DROPPED THROUGH THE DEVICE

1C) AS LONG AS INTRINSIC REGION IS FULLY DEPLETED, $x_p, x_n \ll X_i$ AND $X_d \cong X_i$

SO $C = \frac{K\epsilon_0}{X_i}$

↳ FORWARD BIAS → LIGHTLY DOPED N REGION NO LONGER FULLY DEPLETED, SO $N_A = 10^{19}$, $N_d = 10^{18}$



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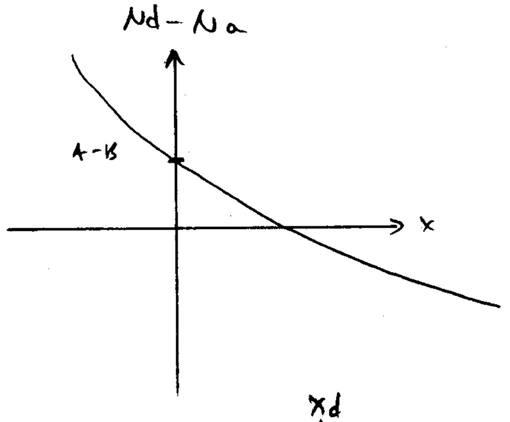
2 a)

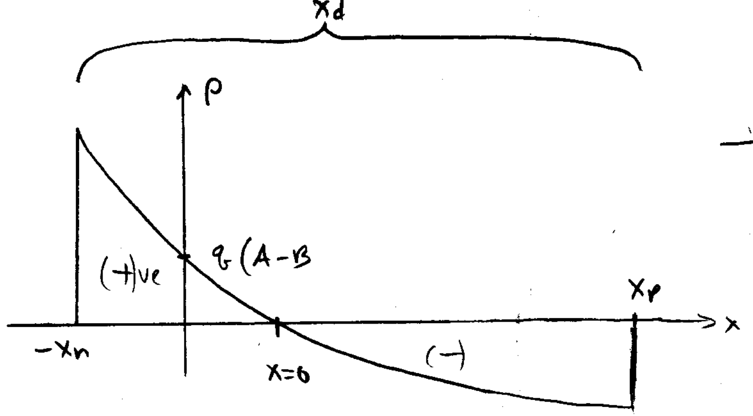


$N_d - N_a = A e^{-\frac{x}{L}} - B$

$\rho(x) = q\,(N_d - N_a)$

$\rho(x) = q\left(A e^{-\frac{x}{L}} - B\right)$

for $-X_n < x < X_p$

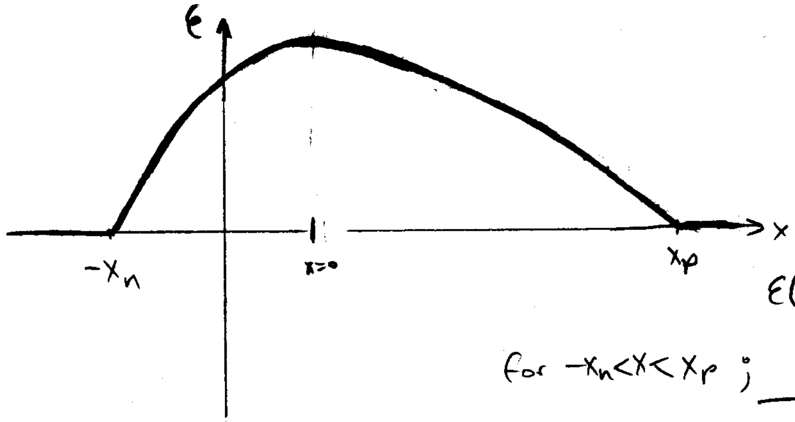


$\mathcal{E}(x) = \int_{-\infty}^{x} \frac{\rho(x)\,dx}{K\epsilon_0} = \frac{q}{K\epsilon_0} \int_{-X_n}^{x} \rho(x)\,dx$

$= \frac{q}{K\epsilon_0} \int_{-X_n}^{x} \left(A e^{-\frac{x}{L}} - B\right) dx$

$= \frac{q}{K\epsilon_0} \left[-LAe^{-\frac{x}{L}} - Bx\right]\Big|_{-X_n}^{x}$

$= -\frac{q}{K\epsilon_0}\Big[\underbrace{BX_n - LAe^{\frac{X_n}{L}}}_{\text{const; } C1} + LAe^{-\frac{x}{L}} + Bx\Big]$

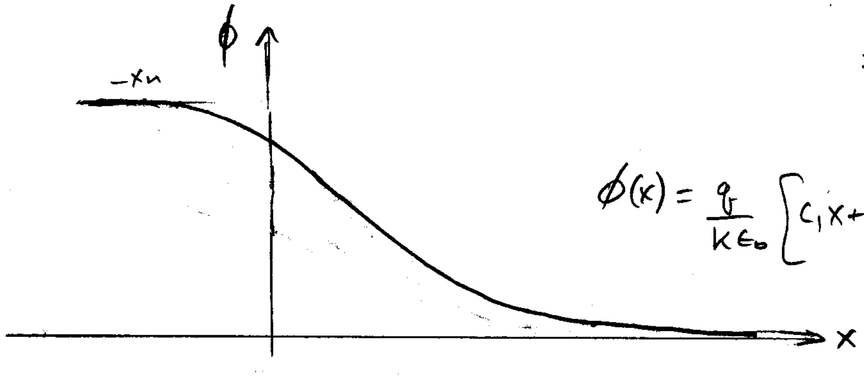


$\mathcal{E}(x) = -\frac{q}{K\epsilon_0}\left[C_1 + LAe^{-\frac{x}{L}} + Bx\right]$

for $-X_n < x < X_p$; $\mathcal{E}(x) = 0$ else

$\phi(x) = -\int_{-\infty}^{x} \mathcal{E}(x)\,dx = \int_{-X_n}^{x} \mathcal{E}(x)\,dx$

$= -\frac{q}{K\epsilon_0}\left[C_1 x + \frac{1}{2}Bx^2 - L^2Ae^{-\frac{x}{L}}\right]_{-X_n}^{x}$



$\phi(x) = \frac{q}{K\epsilon_0}\left[C_1 x + \frac{1}{2}Bx^2 - L^2Ae^{-\frac{x}{L}} - C_2\right]$

where $C_2 = C_1X_n + \frac{1}{2}BX_n^2 - L^2Ae^{\frac{X_n}{L}}$

3/

2b)

$$\phi_i = \frac{kT}{q} \ln \frac{\overset{\text{at } X_p}{N_a} \overset{\text{at } -X_n}{N_d}}{n_i^2} = \frac{kT}{q} \ln \left[\frac{(B - Ae^{-X_p/L})(Ae^{X_n/L} - B)}{n_i^2} \right]$$

and previously

$$(*) \quad \phi_i = \frac{q}{K\epsilon_0}\left(L^2 A e^{-x/L} + \frac{1}{2} B x^2 + C_1 X - C_2\right)$$

To determine X_n & X_p use to find $X_d = X_n + X_p$ use:

$$\int_{-X_n}^{X_p} \rho(x) = \phi$$

$$\text{eq (1):} \quad LAe^{-X_p/L} + BX_p - LAe^{X_n/L} + BX_n = \phi$$

and also

$$\text{eq (2):} \quad \phi_i = -\int_{-X_n}^{X_p} \mathcal{E}(x)\,dx = \frac{q}{K\epsilon_0} C_1 X_p + \frac{1}{2} B X_p^2 - L^2 A e^{-X_n/L}$$

(because X_n term cancelled the C_2 term)

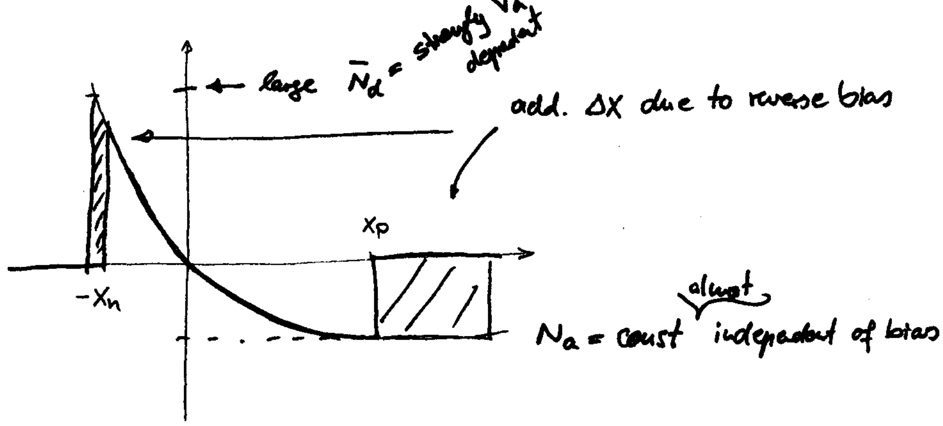
Solve eq (1) and (2) for two unknows X_n & X_p

and ϕ_i can be solved using eq (*)

2)

c) capacitance variation with applied large reverse bias ($|V_a| >> \phi_i$)

$$C' = \frac{k\varepsilon_0}{X_d}$$ (see lecture notes)



$$\Delta X_n \cdot \bar{N}_d = \Delta X_p \cdot N_a$$ (charge neutrality)

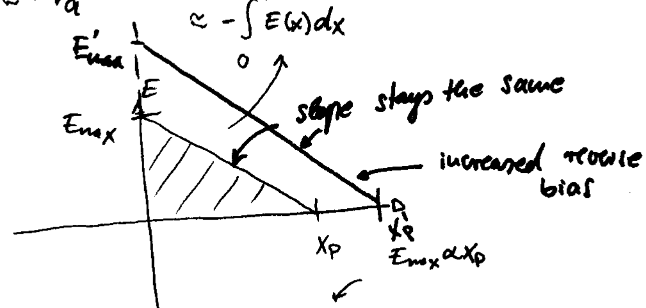
$$\Delta X_n = \underbrace{\frac{N_a}{\bar{N}_d}}_{\text{really small number is limit}} \cdot \Delta X_p$$

$$\Rightarrow \quad X_d = \underbrace{X_n + \Delta X_n}_{\text{small}} + X_p + \Delta X_p \cong X_p$$

$$\Rightarrow \quad C' = \frac{k\varepsilon_0}{X_p}$$

Now need to find $X_p(V_a)$

$$\underbrace{\phi_i - V_a}_{\approx -V_a} = \underbrace{-\int_{-X_n}^{X_p} E(x)\,dx}_{\approx -\int_0^{X_p} E(x)\,dx}$$



$$-V_a = \frac{E_{max}}{2} \cdot X_p$$

$$\Rightarrow -V_a \propto X_p^2$$

$$\frac{1}{X_p} \propto (-V_a)^{-1/2}$$

$$\Rightarrow C' \propto \frac{1}{\sqrt{|V_a|}}$$

3)

Calculate breakdown voltage at $T = 300K$ of an abrupt silicon diode

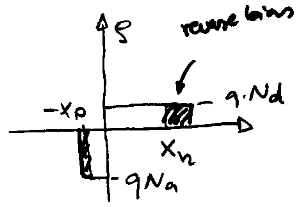
a) $N_d = 10^{19}\,cm^{-3}$ $\qquad W_n = 200\,\mu m$

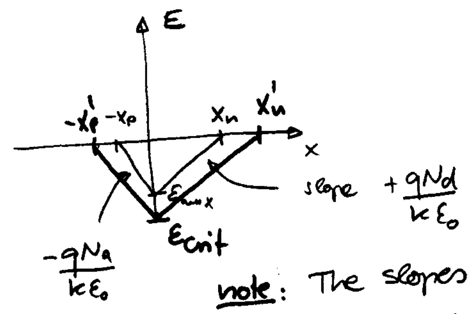
$N_a = 10^{16}\,cm^{-3}$ $\qquad W_p = 0.5\,\mu m$

i) Avalanche breakdown

⇒ need to take $\mathcal{E}_{crit}$ of lighter doped region since most of high field region is on that side (controls slope of $\mathcal{E}$ and scattering lifetime)

Plot lecture notes: $\quad \mathcal{E}_{crit}(10^{16}\,cm^{-3}) = 4\cdot 10^5\,V/cm$





note: The slopes don't change.
⇒ if you know $\mathcal{E}_{crit}$ ⇒ x_n', x_p'

x_p': $\quad \mathcal{E}(x) = \mathcal{E}_{crit} - \frac{qN_a}{k\varepsilon_0}x$

x_n': $\quad \mathcal{E}(x) = \mathcal{E}_{crit} + \frac{qN_d}{k\varepsilon_0}x$

also: $\quad \mathcal{E}(x_n') = 0 \quad \Rightarrow \quad x_n' = -\frac{\mathcal{E}_{crit}k\varepsilon_0}{qN_d} = \frac{|\mathcal{E}_{crit}|k\varepsilon_0}{qN_d}$

$\mathcal{E}(-x_p') = 0 \quad \Rightarrow \quad x_p' = -\frac{\mathcal{E}_{crit}k\varepsilon_0}{qN_a} = \frac{|\mathcal{E}_{crit}|k\varepsilon_0}{qN_a}$

plug-in numbers: $\quad x_n' = 2.58\cdot 10^{-7}\,cm = 2.58\,nm$

$x_p' = 2.58\cdot 10^{-4}\,cm = 2.58\,\mu m$

$\phi_i - V_a = \underbrace{-\int \mathcal{E}(x)dx}_{\text{area under curve}} = \frac{1}{2}\overset{\mathcal{E}_{crit}}{\mathcal{E}_{max}}\cdot(x_n' + x_p') = 51.65V$ $\qquad \phi_i = \frac{kT}{q}\ln\left(\frac{N_aN_d}{n_i^2}\right) = 0.898V$

⇒ $-V_a = 50.752eV$

3)

a)

ii) Zener breakdown

Doesn't happen at $N_a = 10^{16}\,cm^{-3}$ $\Rightarrow$ both doping concentrations need to be at least $\approx 10^{18}\,cm^{-3}$ (see plot)

Basically X_d is too large if you have lightly doped region.

iii) Punch-through breakdown

$$X_n' = 2.58\,nm \quad \ll W_n = 200\,\mu m$$
$$X_p' = 2.58\,\mu m \quad > W_p = 0.5\,\mu m$$

$\Rightarrow$ breakdown will happen via punch-through

$$X_p' = 0.5\,\mu m \rightarrow \phi_i - V_a$$

$$X_p' = \left(\frac{2K\varepsilon_0}{q}(\phi_i - V_a)\frac{N_d}{N_a(N_a+N_d)}\right)^{1/2}$$

$$\Rightarrow \quad X_p'^2 \cdot \frac{N_a(N_a+N_d)}{N_d} \cdot \frac{q}{2K\varepsilon_0} = \phi_i - V_a$$

$$-V_a = X_p'^2 \cdot \frac{N_a(N_a+N_d)}{N_d} \frac{q}{2K\varepsilon_0} - \phi_i$$

$$-V_a = 1.935 - 0.898\,V = 1.037\,V$$

breakdown voltage is: $\boxed{-V_a = 1.037\,V}$

3)

b) $N_d = 10^{19}\ cm^{-3}$ $\quad W_n = 200\mu m$

$N_a = 10^{18}\ cm^{-3}$ $\quad W_p = 0.5\mu m$

i) Avalanche breakdown

$\mathcal{E}_{crit}(10^{18} cm^{-3}) = 1.3\,10^6\ \frac{V}{cm}$ (plot lecture notes)

analysis equivalent to a)

plug-in numbers: $x_n' = \sqrt[0.65]{1.293\cdot 10^{-6}}\ cm = \sqrt[0.65]{1.293\cdot 10^{-2}}\ \mu m = 8.4\ nm$

$x_p' = \sqrt[0.65]{1.293\cdot 10^{-5}}\ cm = \sqrt[0.65]{1.293\cdot 10^{-1}}\ \mu m = 84\ nm$

$\phi_i = \frac{kT}{q}\ln\left(\frac{N_a N_d}{n_i^2}\right) = 1.02V$

$\phi_i - V_a = \frac{1}{2}\mathcal{E}_{crit}(x_n' + x_p') = 8.04\ V$

$-V_a = 7.02V$

ii) Punch-through breakdown

$\left.\begin{array}{l} x_n' \ll W_n \\ x_p' < W_p \end{array}\right\} \Rightarrow$ no punch-through

iii) Zener breakdown

$\mathcal{E}_{crit}^{zener} = 1.4\cdot 10^6\ V/cm \cong \mathcal{E}_{crit}^{avalanche}$

Note: ~~Zener breakdown occurs at a slightly higher critical field than avalanche.~~

Breakdown will occur via both mechanisms, with avalanche dominant

8/

$\boxed{4}$ $N_a = 10^{19}\ cm^{-3}$ $\quad N_d = 10^{17}\ cm^{-3}$ $\quad \tau_n = \tau_p = 0.5\ \mu s$

$W_p = 0.1\ \mu m$ ($1\times10^{-7}\ m$) $\quad W_n = 500\ \mu m$ $\quad V_a = 0.6\ V$ (FW biased)

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.93146\ V$$

$$x_n = \sqrt{\frac{2\epsilon V_{bi}}{q}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)} = 1.0918\times10^{-5}\ cm$$

$$x_p = \sqrt{\frac{2\epsilon V_{bi}}{q}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)} = 1.0918\times10^{-7}\ cm$$

$$D_n = \frac{kT}{q}\mu_n = 34.9$$

$$D_p = \frac{kT}{q}\mu_p = 12.41$$

$$L_n = \sqrt{D_n\tau_n} = 4.1773\times10^{-3}\ cm$$

$$L_p = \sqrt{D_p\tau_p} = 2.49\times10^{-3}\ cm$$

$$n_{p_o} = n_i^2/N_a = 22.5\ cm^{-3}$$

$$p_{n_o} = n_i^2/N_d = 2.25\times10^{3}\ cm^{-3}$$

$W_p < L_n \Rightarrow$ Short diode

$$p_n(x_n) = p_{n_o}\exp(qV_a/kT) = 2.70195\times10^{13}\ cm^{-3}$$

$$n_p(x_p) = n_{p_o}\exp(qV_a/kT) = 2.70195\times10^{11}\ cm^{-3}$$

$J_n(x) = q\dfrac{D_n n_{p_o}}{W_p}\left[\exp\left(\dfrac{qV_a}{kT}\right)-1\right] =$ [redacted]

$J_n(W_p) = 0.15108\ A/cm^2$

a) $J_p(x_n) = q\dfrac{D_p p_{n_o}}{L_p}\left[\exp(qV_a/kT)-1\right] = \boxed{2.1525\times10^{-2}\ A/cm^2}$

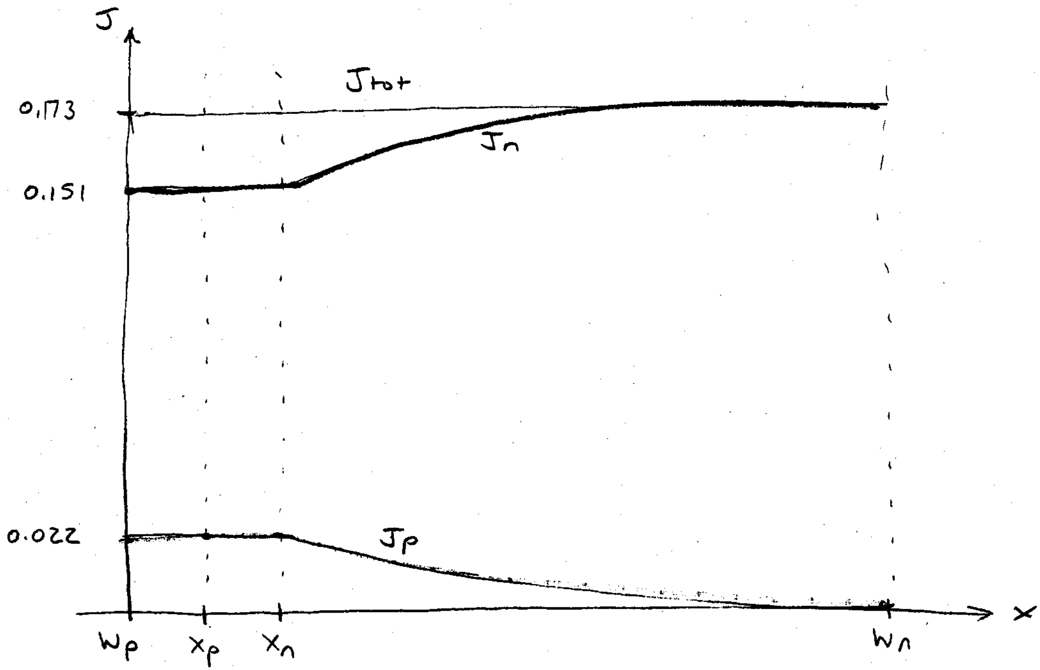
$J_{tot} = J_p(x_n) + J_n(x_p) = 0.17265\ A/cm^2$

b) $J_p(x_p) = J_p(x_n) = \boxed{2.157\times10^{-2}\ A/cm^2}$ (no recombination)

$\boxed{4}$

c) $J_n(W_p) = q\dfrac{D_n n_{po}}{W_p}\left[\exp\left(\dfrac{qV_a}{KT}\right) - 1\right] = \boxed{0.15108\ A/cm^2}$

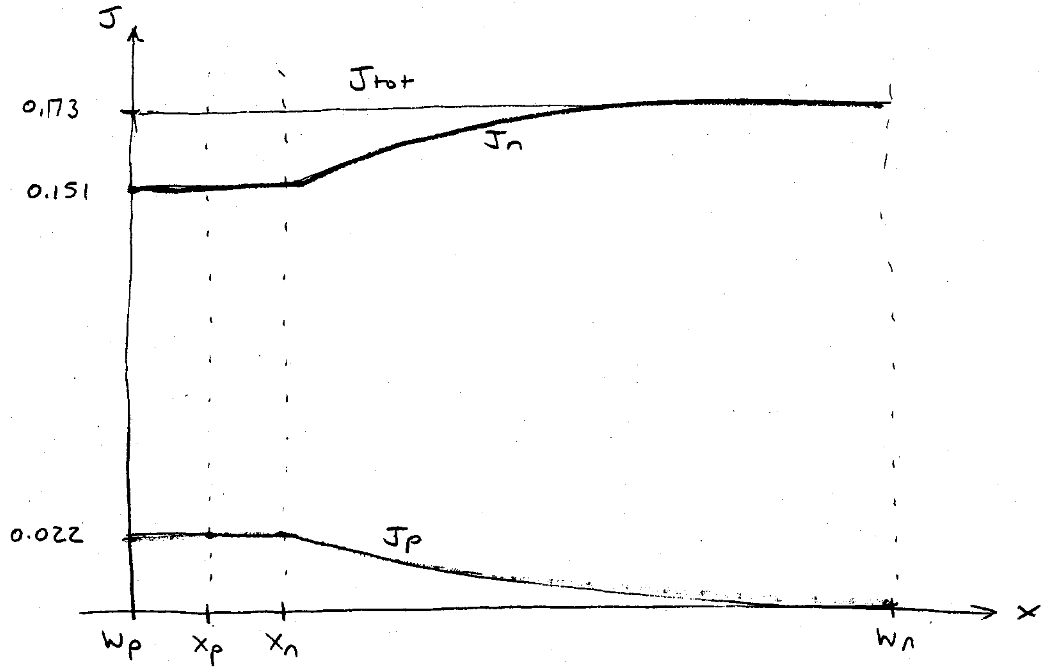
d) $J_n(W_n) \approx J_{tot} = J_p(x_n) + J_n(W_p) = \boxed{0.1726\ A/cm^2}$



$\boxed{4}$

c) $J_n(W_p) = q\frac{D_n n_{po}}{W_p}\left[\exp\left(qV_a/kT\right) - 1\right] = \boxed{0.15108\ A/cm^2}$

d) $J_n(W_n) \approx J_{tot} = J_p(x_n) + J_n(W_p) = \boxed{0.1726\ A/cm^2}$



$$p = \frac{10^{19}}{2} + \sqrt{\left(\frac{10^{19}}{2}\right)^2 + (1.5\times10^{10})^2} = 10^{19}$$

$$n = \frac{10^{19}}{n_i^2} = 0.044\ cm^{-3} = n_{po}$$

$N_a = 10^{19}$ $N_d = 10^{17}$

$$\rightarrow n = 10^{17},\ p = \frac{10^{17}}{n_{i2}} = 4.44\times10^{-4}\ cm^{-3} = p_{no}$$

5a)

$$x_n = \left[\frac{2K\varepsilon_0}{q}\phi_i\left(\frac{N_a}{N_d(N_a+N_d)}\right)\right]^{\frac{1}{2}} = \left[\frac{2(11.7)(8.854\times10^{-14})}{(1.6\times10^{-19})}(0.937)\left(\frac{10^{19}}{10^{17}(10^{19}+10^{17})}\right)\right]^{\frac{1}{2}}$$

$$= 1.096\times10^{-5}\ cm$$

$$x_p = \left[\frac{2K\varepsilon_0}{q}\phi_i\left(\frac{N_d}{N_a(N_a+N_d)}\right)\right]^{\frac{1}{2}} = 1.096\times10^{-7}\ cm = 1.096\times10^{-3}\ \mu$$

$$x_d = x_n + x_p = 1.107\times10^{-5}\ cm$$

$$\Delta p = p_{no}\left(\exp\frac{qV_a}{kT} - 1\right)\exp - \frac{x - x_n}{L_p}$$

@ $x = x_n$ $\quad \Delta p = (4.44\times10^{-4})\left(\exp\frac{0.6}{0.026} - 1\right)\cancel{\exp - \frac{x_n - x_n}{L_p}} = 4.673\times10^{6}\ cm^{-3}$

GENERAL: $\Delta p = 4.673\times10^{6}\ \exp\left(-\frac{x - x_n}{0.00180}\right)$ $\quad x > x_n$

@ $x = x_p$ $\quad \Delta n = (0.044)\left(\exp\frac{0.6}{0.026} - 1\right)\exp\frac{x + x_p}{L_n} = 4.631\times10^{8}\ cm^{-3}$

GENERAL: $\Delta n = 4.631\times10^{8}\ \exp\left(\frac{x + x_p}{0.00127}\right)$ $\quad x < -x_p$

B)

$$I_p = qA\frac{D_p p_{no}}{L_p}\left(\exp\frac{qV_a}{kT} - 1\right)\exp - \frac{x - x_n}{L_p} = \frac{(1.6\times10^{-19})(1.3)(4.44\times10^{-4})(A)}{0.00180}\left(\exp\frac{0.6}{0.026} - 1\right)\exp - \frac{x - x_n}{L_p}$$

$$I_p = 5.4\times10^{-10}(A)\exp - \frac{x - x_n}{0.00180}$$

$$I_n = qA\frac{D_n n_{po}}{L_n}\left(\exp\frac{qV_a}{kT} - 1\right)\exp\frac{x + x_p}{L_n} = \frac{(1.6\times10^{-19})(20.8)(0.044)(A)}{0.00127}\left(\exp\frac{0.6}{0.026} - 1\right)\exp\frac{x + x_p}{L_n}$$

$$I_n = 1.213\times10^{-6}(A)\exp\frac{x + x_p}{0.00127}$$

$$J_n = \frac{I_n}{A}$$

5 a)

$$p_n(x) = p_{n0} + p_{n0}\, e^{x_n/L_p} \left(e^{qV_a/kT} - 1\right) e^{-x/L_p} \qquad p_{n0} = \frac{n_i^2}{N_d}$$

$$n_p(x) = Ax + B \quad \text{where} \quad \begin{cases} A = \dfrac{n_{p0}\left(e^{qV_a/kT} - 1\right)}{W_p - x_p} \\ \\ B = n_{p0} + AW_p \end{cases} \qquad n_{p0} = \frac{n_i^2}{N_a}$$

b) General form for Current Density:

$$J_p(x) = -q D_p\, p_{n0}\, e^{x_n/L_p} \left(e^{qV_a/kT} - 1\right)\left(-\frac{1}{L_p}\right) e^{-x/L_p}$$

$$J_{np}(x) = q D_n \frac{n_i^2}{N_a} \frac{e^{qV_a/kT} - 1}{W_p - x_p}$$

5 c) Calculate the drift and diffusion components of the majority currents.

$$J^{n}_{drift} = -q\mu_n n(x)\, \mathcal{E}(x) \qquad J^{p}_{drift} = q\mu_p p(x)\, \mathcal{E}(x)$$

$$J^{n}_{diff} = (-q)\underbrace{\left(-D_n \frac{dn}{dx}\right)}_{\text{flux}} \qquad J^{p}_{diff} = q\underbrace{\left(-D_p \frac{dp}{dx}\right)}_{\text{flux}}$$

$$J^{n}_{diff} = q D_n \frac{dn}{dx}$$

n-region:

quasi neutrality $\Delta n_n = \Delta p_n$

$$n_n(x) = \underbrace{n_{n_0}}_{\text{constant}} + \Delta n_n(x) \overset{\swarrow}{=} n_{n_0} + \Delta p_n(x)$$

$$\text{i)} \Rightarrow J^{n_n}_{diff}(x) = q\cdot D_n \cdot \frac{dn_n}{dx} = q\cdot D_n \frac{d\Delta p_n}{dx}$$

from 4) $\Delta p_n(x) = p_n(x) - p_{n_0} = p_{n_0}\, e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)e^{-x/L_p}$

$$\Rightarrow J^{n_n}_{diff}(x) = q\cdot D_n\cdot p_{n_0}\, e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)\cdot\left(-\frac{1}{L_p}\right)\cdot e^{-x/L_p}$$

$$\text{ii)} \Rightarrow J^{n_n}_{drift} = -q\mu_n n_n(x)\,\mathcal{E}(x) = -q\mu_n n_{n_0}\,\mathcal{E}(x)$$

$n_n(x) \approx n_{n_0}$ (low level injection)

p-region:

quasineutrality $\Delta p_p = \Delta n_p$

$$p_p(x) = p_{p_0} + \Delta p_p(x) \overset{\swarrow}{=} p_{p_0} + \Delta n_p(x)$$

p_{p_0}: constant

$$\text{i)} \Rightarrow J^{p_p}_{diff}(x) = -q D_p \frac{dp_p}{dx} = -q D_p\cdot\frac{d\Delta n_p(x)}{dx}$$

$$A = \frac{n_{p_0}\left(e^{\frac{qV_a}{kT}}-1\right)}{W_p - x_p}$$

$$B = n_{p_0} + A W_p$$

from 4) $\Delta n_p(x) = n_p(x) - n_{p_0} = A\cdot x + B - n_{p_0}$

$$J^{p_p}_{diff}(x) = -q D_p\cdot A = -\frac{q D_p\cdot n_{p_0}\left(e^{\frac{qV_a}{kT}}-1\right)}{W_p - x_p}$$

5 c)

ii) $J^{p_p}_{drift} = q\mu_p p_p(x)\, \mathcal{E}(x)$

$J^{p_p}_{drift} = q\mu_p p_{p_0}\, \mathcal{E}(x)$

$p_p(x) \approx p_{p_0}$ (low level injection)

(1) $J_{Total} = J^{pn}_{diff} + J^{nn}_{diff} + J^{nn}_{drift}$

(2) $J_{Total} = J^{pn}_{diff} + J^{np}_{diff}(-x_p) = \text{const.}$

(1) = (2) $\Longrightarrow$ $\boxed{J^{nn}_{drift} = J_{Total} - J^{pn}_{diff} - J^{nn}_{diff}}$

5 d) calculate $\mathcal{E}(x)$ in n and p region outside the depletion region

n-region:

$J^{n_n}_{drift} = -q\mu_n n_{n_0}\, \mathcal{E}(x)$

$$-q\mu_n n_{n_0}\mathcal{E}(x) = J_{total} - \frac{qD_p p_{n_0}}{L_p} e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)e^{-x/L_p} + q\frac{D_n p_{n_0}}{L_p} e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)e^{-x/L_p}$$

$$\mathcal{E}(x) = -\frac{1}{q\mu_n n_{n_0}}\left(J_{total} + \frac{q p_{n_0}}{L_p} e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)(D_n - D_p)\, e^{-x/L_p}\right)$$

p-region:

$J^{p_p}_{drift} = q\mu_p p_{p_0}\, \mathcal{E}(x)$

(1) J_{total} = const (see above)

(2) $J_{total} = J^{n_p}_{diff} + J^{p_p}_{diff} + J^{p_p}_{drift}$

$\Longrightarrow$ $\boxed{J^{p_p}_{drift} = J_{total} - J^{n_p}_{diff} - J^{p_p}_{diff}}$

$$q\mu_p p_{p_0}\mathcal{E}(x) = J_{total} - \frac{qD_n n_{p_0}\left(e^{\frac{qV_a}{kT}}-1\right)}{W_p - x_p} + \frac{qD_p n_{p_0}\left(e^{\frac{qV_a}{kT}}-1\right)}{W_p - x_p}$$

$\Longrightarrow \mathcal{E}(x) = \text{const}$

5 e) Diff. approximation

$J_{diff}^{minority}$ neglectable

check

$J_{drift}^{pn} \ll J_{diff}^{pn}$

n-region:

$$\boxed{J_{drift}^{p_n} = q\mu_p p_n(x)\,\mathcal{E}(x)}$$

$$\mathcal{E}(x) = \frac{1}{q\mu_n n_{n_0}}(\ldots)$$

quasi-neutrality

very small

$$\Rightarrow J_{drift}^{p_n} \propto \frac{p_n(x)}{n_{n_0}} \approx \underbrace{\frac{\Delta p_n}{n_{n_0}}}$$

✓ low-level injection assumption

p-region:

$$J_{drift}^{n_p} = q\mu_n n_p(x)\,\mathcal{E}(x)$$

$$\mathcal{E}(x) = \frac{1}{q\mu_p p_{p_0}}(\ldots)$$

$$\Rightarrow J_{drift}^{n_p} \propto \frac{n_p(x)}{p_{p_0}} \approx \underbrace{\frac{\Delta n_p}{p_{p_0}}}_{\text{very small}}$$

(low level injection assumption)

→ if low level injection is ok diff. approx. is ok.

5 f) use currents to get charge densities as a function of position

idea: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{k\varepsilon_0}$

1-D $\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{k\varepsilon_0}$

n-region:

$$\mathcal{E}(x) = -\frac{1}{q\mu_n n_{no}}\left(J_{total} + \frac{q p_{no}}{L_p} e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)(D_n - D_p)\, e^{-x/L_p}\right)$$

$$\frac{d\mathcal{E}}{dx} = -\frac{1}{\cancel{q}\mu_n n_{no}} \cdot \frac{\cancel{q} p_{no}}{L_p} e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)(D_n-D_p)\cdot\left(-\frac{1}{L_p}\right)e^{-x/L_p}$$

$$= \frac{p_{no}}{n_{no} L_p^2/\mu_n} e^{x_n/L_p}\left(e^{\frac{qV_a}{kT}}-1\right)(D_n-D_p)\, e^{-x/L_p} \overset{!}{=} \frac{\rho(x)}{k\varepsilon_0}$$

$$\left.\begin{aligned} p_{no} &= \frac{n_i^2}{N_d} \\ n_{no} &= N_d \end{aligned}\right\} \Rightarrow \rho(x) = k\varepsilon_0 \left(\frac{n_i}{N_d}\right)^2 \cdot \frac{e^{x_n/L_p}}{L_p^2/\mu_n}\left(e^{\frac{qV_a}{kT}}-1\right)(D_n-D_p)\, e^{-x/2_p}$$

p-region:

$\mathcal{E}(x) = \text{const}$

$\Rightarrow \frac{d\mathcal{E}}{dx} = 0 \quad \Rightarrow \rho(x) = 0$

Quasi-neutrality assumption: $|\rho(x)| \ll |q(N_a - N_d)|$:

$\rho(x) = 0$

in p-region: completely fullfilled $\rho(x) = 0$

in n-region: $\rho(x) \propto \left(\frac{n_i}{N_d}\right)^2$

here: $n_i \ll N_d \Rightarrow \left(\frac{n_i}{N_d}\right)^2$ very small

$\rightarrow$ fullfilled up to a term $\propto \left(\frac{n_i}{N_d}\right)^2$