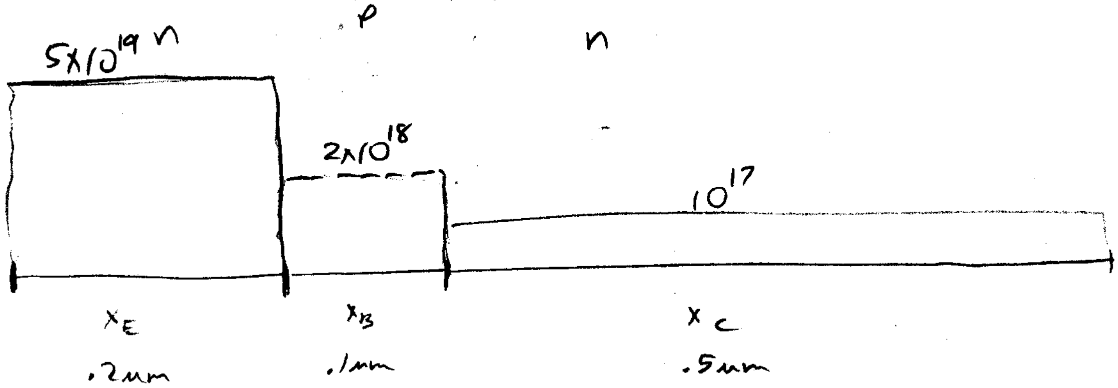
EE 482 HW # 8 Solutions



1.a)

(eq 30)

$$\alpha_F = \frac{I_C}{I_E} = \alpha_T \gamma \delta$$

From plot in notes:

$D_{pE} \cong 7.29 \frac{cm^2}{s}$; $D_{nB} = 5.18 \frac{cm^2}{s}$; $D_{pC} = 6.47 \frac{cm^2}{s}$

$$L_{nB} = \sqrt{D_{nB} \tau_{nB}} = (5.18 \times 0.5 \times 10^{-6} s)^{1/2} \quad \text{all } \tau\text{'s} = .5\mu s$$

$$= 1.6 \times 10^{-3} \text{ cm} \left(\frac{1 \mu m}{10^{-4} cm}\right) = 16.1 \mu m$$

$$\alpha_T \cong 1 - \frac{x_B^2}{2 L_{pB}^2} \qquad x_B \ll L_{nB} \quad \text{ok}$$

$$= 1 - \frac{(.1 \mu m)^2}{2 (16.1)^2} = .99998 \cong 1 \implies (\alpha_F \cong \gamma_F)$$

$$\alpha_F = \gamma_F = \frac{1}{1 + \frac{x_B}{x_E} \frac{N_{aB}}{N_{dE}} \frac{D_{pE}}{D_{nB}}} = \frac{1}{1 + \frac{.1 \mu m}{.2 \mu m} \left(\frac{2 \times 10^{18}}{5 \times 10^{19}}\right) \frac{7.29}{5.18}}$$

$$\alpha_F = .99504$$

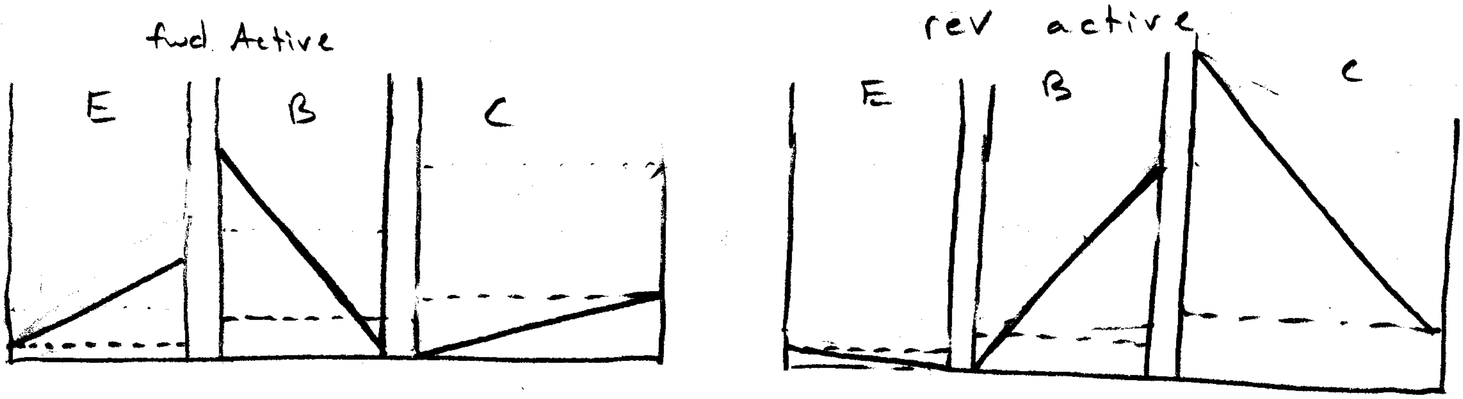
$$\beta_F = \frac{\alpha_F}{1 - \alpha_F} = 201$$

1. b) $\alpha_R = \alpha_T \gamma_R \cong \gamma_R$

$$\gamma_R = \frac{1}{1 + \frac{x_B}{x_C}\frac{D_{pC}}{D_{pB}}\frac{N_{aB}}{N_{dC}}} = \frac{1}{1 + \frac{.1}{.5}\left(\frac{6.47}{5.18}\right)\frac{2\times10^{18}}{10^{17}}}$$

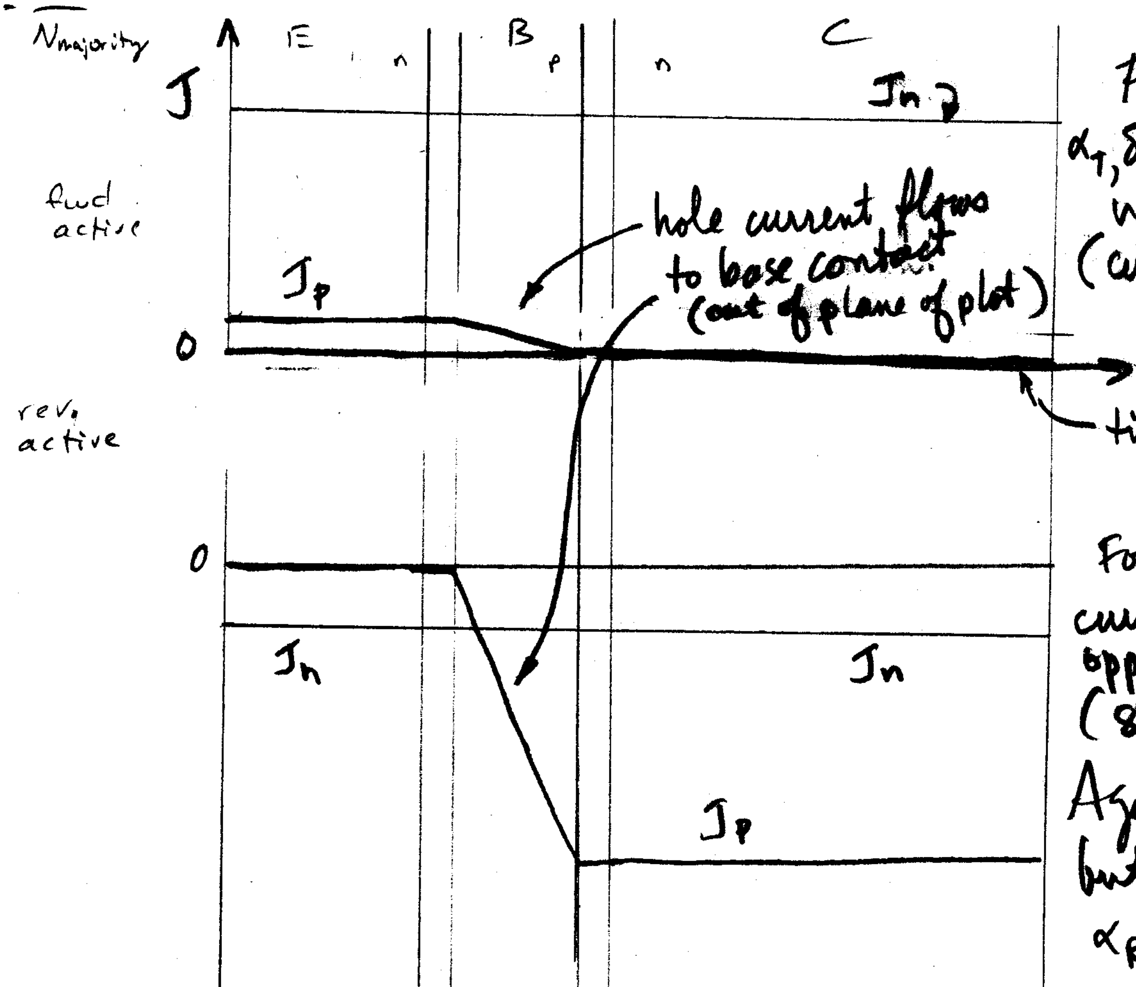
$\alpha_R = \gamma_R = .1668 \quad \Rightarrow \beta_R = \frac{\alpha_R}{1-\alpha_R} = .2$

c) - Minority Charge Densities -



- Current Densities -

$N_{minority} = \frac{n_i^2}{N_{majority}}$



For forward active $\alpha_T, \delta \cong 1$, so J_n nearly constant (current from C to E)

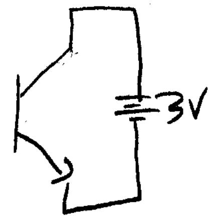
For reverse active, current flows in opposite direction (shown as negative)

Again $J_n \doteq$ constant, but $J_p <> J_n$ since $\alpha_R \ll 1$

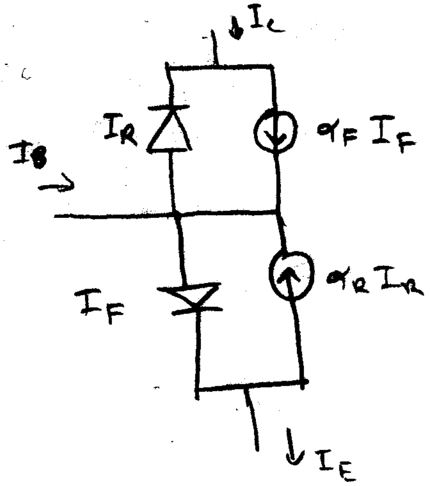
(a) npn $\alpha_F = .995$ $\alpha_R = .2$

$V_{CE} = 3V$; $I_B = 1\mu A$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = 199$$



Ebers-Moll Model



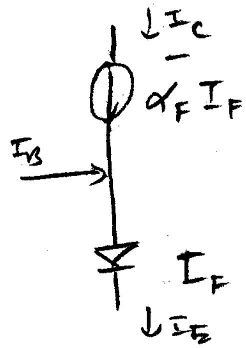
$V_{CE} = 3V$

$V_{CB} > 0$

$V_{BE} > 0$

fwd active

since it is fwd active neglect reverse currents



$I_E = I_B + I_C$

$I_C = I_F - I_B \overset{\text{also}}{=} \alpha_F I_F$

$I_B = (1-\alpha_F) I_F \Rightarrow I_F = \frac{I_B}{1-\alpha_F} = 200\mu A$

$I_C = (.995)\, 200\mu A$

$\underline{I_C = 199\mu A}$

Also we can find this by using

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = 199$$

$$I_C = \beta_F I_B = 199\mu A$$

2b)

$V_{CE} = ?$

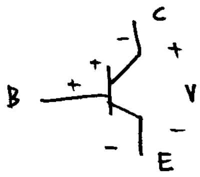
$I_B = 10\,\mu A$

$I_C = 100\,\mu A$

$\alpha_F = .995 \qquad \beta_F = \frac{\alpha_F}{1-\alpha_F} = 199 \qquad \alpha_R = .2$

expected $I_C = \beta I_B = (199)(10\,\mu A) \approx 2\,mA$

but $I_C \ll 2\,mA$ so there must be loss due to significant reverse current.

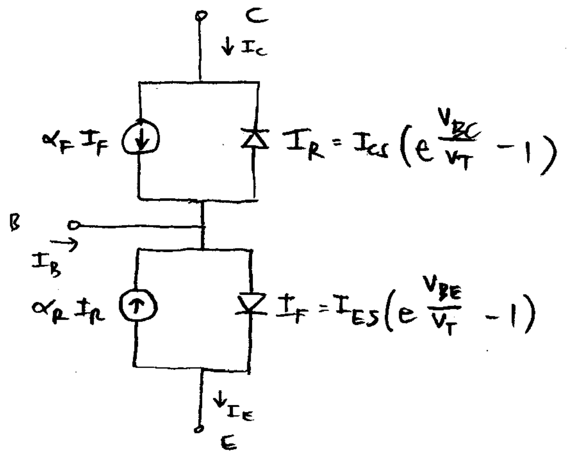


$V_{CE\,SAT}$ $\quad V_{BE} > 0$ because I_B is positive.

$V_{BC} > 0$ because of reverse current.

SATURATION MODE

Ebers - Moll Model



(1) $I_C + I_B = I_E$

(2) $I_C = \alpha_F I_F - I_R$

(3) $I_E = I_F - \alpha_R I_R$

(2) $I_R = \alpha_F I_F - I_C$

(3) $\cancel{\alpha_R} I_R = \frac{I_F - I_E}{\alpha_R}$

$\Rightarrow \alpha_F I_F - I_C = \frac{I_F - I_E}{\alpha_R} \Rightarrow \alpha_R \alpha_F I_F - \alpha_R I_C = I_F - I_E \Rightarrow$

$I_F(\alpha_R \alpha_F - 1) = \alpha_R I_C - I_E \Rightarrow I_F = \frac{\alpha_R I_C - I_E}{(\alpha_R \alpha_F - 1)}$

$I_F = \frac{(.2)(100\,\mu A) - (100\,\mu A + 10\,\mu A)}{(.2)(.995) - 1} = 112.4\,\mu A$

$I_R = \alpha_F I_F - I_C$

$= (.995)(112.4\,\mu A) - (100\,\mu A)$

$= 11.8\,\mu A$

$I_R = I_{CS}\left(e^{\frac{V_{BC}}{V_T}} - 1\right) \Rightarrow V_{BC} = V_T \ln\left(\frac{I_R}{I_{CS}} + 1\right) \Rightarrow V_{BC} = V_T \ln\left(\frac{I_R}{I_{CS}}\right)$

$I_F = I_{ES}\left(e^{\frac{V_{BE}}{V_T}} - 1\right) \Rightarrow V_{BE} = V_T \ln\left(\frac{I_F}{I_{ES}} + 1\right) \Rightarrow V_{BE} = V_T \ln\left(\frac{I_F}{I_{ES}}\right)$

ASSUME $I_R \gg I_{CS}$ and $I_F \gg I_{CS}$

continued →

$$V_{CE} = V_{BE} - V_{BC}$$

2b) cont.

$$= V_T \ln\left(\frac{I_F}{I_{ES}}\right) - V_T \ln\left(\frac{I_R}{I_{CS}}\right)$$

$$= V_T\left[\ln(I_F) - \ln(I_{ES}) - \ln(I_R) + \ln(I_{CS})\right]$$ (identity for natural logs)

$$= V_T\left[\ln(I_F) - \ln(I_R) - \ln(I_{ES}) + \ln\left(\frac{\alpha_F}{\alpha_R} I_{ES}\right)\right]$$ (reciprocity $\alpha_F I_{ES} = \alpha_R I_{CS}$

$$= V_T\left[\ln(I_F) - \ln(I_R) - \cancel{\ln(I_{ES})} + \ln\left(\frac{\alpha_F}{\alpha_R}\right) + \cancel{\ln(I_{ES})}\right]$$ (identity)

$$=(.026)\left[\ln(112.4\,mA) - \ln(11.8\,mA) + \ln\left(\frac{.995}{.2}\right)\right]$$

$$= 0.10 \text{ V}$$

Check using eqn. (41) [pg 18]

$$V_{CEsat} \doteq (.026)\ln\left(\frac{\left[1+\frac{100mA}{10mA}(1-.2)\right]}{(.2)\left[1-\frac{100mA}{10mA}\left(\frac{1-.995}{.995}\right)\right]}\right)$$

$$= (.026)(3.86)$$

$$= \underline{\underline{0.10 \text{ V}}}$$ ok.