3. A silicon p-channel (n-substrate) MOS capacitor with a polysilicon gate heavily doped with boron ($E_f = E_v$) has a substrate doping of $N_d = 2 \times 10^{18}$ cm$^{-3}$ and an oxide thickness of $x_{ox} = 2$ nm. There are no significant interface charges. The device is biased such that the voltage dropped across the oxide is 0.5 V (more positive toward gate).

(a) Calculate the charge in the semiconductor and on the gate. What mode is the device operating in? (10)

$$V_{ox} = \frac{Q_g}{C_{ox}} = -\frac{Q_s}{C_{ox}} \quad \text{(no oxide charges)}$$

$$C_{ox} = \frac{k_{ox} \varepsilon_0}{x_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14} \, \text{F/cm}}{2 \times 10^{-7} \, \text{cm}} = 1.73 \times 10^{-6} \, \text{F/cm}^2$$

$$Q_g = 0.5 \, \text{V} \times 1.73 \times 10^{-6} \, \text{F/cm}^2 = 8.6 \times 10^{-7} \, \text{C/cm}^2$$

$$Q_s = -Q_g = -8.6 \times 10^{-7} \, \text{C/cm}^2 < 0 \Rightarrow \text{accumulation of electrons in n-substrate}$$

(b) What is the applied voltage between the gate and substrate? (12)

$$V_{GB} = V_{FB} + V_S + \Delta V_{ox} = \phi_{MS} + V_S + V_{ox}$$

$$\phi_M = \chi_S + E_g, \quad \phi_S = \chi_S + \frac{kT}{q} \ln \frac{N_c}{N_d} = \chi_S + 0.07 \, \text{V}$$

$$\phi_{MS} = 1.12 - 0.07 = 1.05 \, \text{V}$$

$$V_S \approx 0 \text{ in accumulation}$$

$$V_{GB} = 1.05 + 0.5 = 1.55 \, \text{V}$$
4. A silicon n-channel MOS transistor has an oxide thickness of 2 nm, uniform substrate doping of \( N_a = 2 \times 10^{18} \text{cm}^{-3} \) and \( W = 300 \text{nm} \) and \( L = 100 \text{nm} \). \( V_T = 0.4 \text{V} \) with \( V_S = V_B = 0 \text{V} \). Assume \( \mu_n = 150 \text{cm}^2/\text{Vs} \) in the inversion layer.

(a) If the gate metal work function is 4.0V, what must the oxide interface charges be (assume no charges in bulk of oxide)? (12)

\[
V_T = V_{FB} - 2 \phi_F - \frac{Q_{d,\text{max}}}{C_{ox}}
\]

\[\phi_F = \phi_p = -\frac{kT}{q} \ln \left( \frac{N_d}{N_i} \right) = 0.495\]

\[\phi_s = \phi_s + \phi_F = -\frac{kT}{q} \ln \left( \frac{N_V}{N_d} \right) = 4.05 + 1.12 - 0.07\]

\[Q_{d,\text{max}} = \sqrt{2 e K_s \epsilon_0 N_d l - 2 e F}\]

\[= 8.14 \times 10^{-7} \text{C/cm}^2\]

(b) Calculate the change in threshold voltage with the change in channel to substrate bias \( \delta \) for \( V_{SB} = 0 \). (12)

\[
\delta = \frac{C_d'}{C_{ox}} = \frac{4.1 \times 10^{-7} \text{F/cm}^2}{1.73 \times 10^{-6} \text{F/cm}^2} = 0.24
\]

\[C_d' = \frac{K_s \epsilon_0}{x_d} = \frac{8.8 \times 10^{-14} \text{F/m}}{2.54 \times 10^{-6} \text{m}} = 2.54 \times 10^{-6} \text{F/cm}^2\]

\[x_d = \frac{Q_{d,\text{max}}}{q N_a} = \frac{8.14 \times 10^{-7} \text{C/cm}^2}{(1.6 \times 10^{-19} \text{C})(2 \times 10^{18} \text{cm}^{-3})} = 2.54 \times 10^{-6} \text{cm}\]

(c) If \( V_S = V_B = 0 \text{V} \) and \( V_G = 2 \text{V} \), what drain voltage would bring the transistor to the edge of saturation. (Use full or linearized model). (8)

\[V_{DS}^{Sat} = V_G - V_T = \frac{2 - 0.4}{1 + 0.24} = 1.29 \text{V}\]
(d) For $V_D = 2V$ (and $V_S = V_B = 0V$, $V_G = 2.0V$), determine the operating mode and calculate the drain current. Include channel length modulation if appropriate. (14)

\[ V_{DS} = 2V > V_{DS}^{Sat} \]
\[ L = L - \Delta L \]
\[ \Delta L = \left[ \frac{2K_S \delta \left(V_{DS} - V_{DS}^{Sat}\right)^{\frac{1}{2}}}{\mu_0 n_{i0} \frac{W}{L}} \right] \frac{2(1.8)(8.85 \times 10^{-14} \text{F/cm})}{(1.6 \times 10^{-9} \text{C})(2 \times 10^{10} \text{cm}^{-3})} \]
\[ = 2.0 \times 10^{-6} \text{cm} \]

\[ I_{DS}^{Sat} = \mu_0 \frac{W}{L} C_\text{ox} \left( \frac{V_{DS}^{Sat} - V_T}{2} \right)^2 \]
\[ = \mu_0 \frac{2}{1+\delta} \left( \frac{150 \text{cm}^2}{V_s} \right) \left( \frac{300}{80} \right) \left( 1.73 \times 10^{-6} \frac{\text{F}}{\text{cm}^2} \right) \]
\[ \times \left( \frac{2-0.4}{2} \right)^2 \frac{1.24}{1+1.24} \]
\[ = 1.0 \times 10^{-3} \text{A} = 1 \text{mA} \]

\[ \Sigma_{off} = \frac{V_{GS} + V_T + 0.2}{6 \text{V/cm}} = 2.2 \times 10^{6} \text{A/cm} \]

\[ \mu_n \approx 90 \text{cm}^2/\text{V.s} \text{ from plot, but use } 150 \text{cm}^2/\text{V.s} \]

(e) Sketch the band diagram in channel near drain if $V_G = 2.0V$, $V_D = 2V$, and $V_B = V_S = 0V$. What is the voltage dropped across the semiconductor? (12)

\[ V_{DS} > V_{DS}^{Sat}, \text{ so drain end of channel not inverted} \]

\[ V_{GB} = V_{FB} + V_S + \sqrt{2K_S \delta q N_A V_S} \]
\[ = \delta_{MS} - \frac{Q_{DS}}{C_\text{ox}} + V_S + 0.47 \sqrt{V_S} \]
\[ 2 = 1.1 + 0.4 + V_S + 0.47 \sqrt{V_S} \]
\[ x = \sqrt{V_S} \]
\[ x^2 + 0.47x - 3.06 = 0 \]
\[ x = -0.47 \pm \sqrt{(0.47)^2 + 4(3.06)} \]
\[ = 1.53 \]
\[ V_S = 2.34 \text{V} \]