Home Work #2 - EE482

1- $f_{MB}^{app} = \exp\left(-\frac{(E-E_f)}{kT}\right)$ for $E-E_f > 4kT$

$f_{FD} = \dfrac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$

$\Bigg\}$: Occupation Probability

$$error = \frac{f_{MB}^{app.} - f_{FD}}{f_{FD}} = \exp\left(-\frac{(E-E_f)}{kT}\right) < \frac{3}{100}$$

$$\Longrightarrow E-E_f > kT\ln\left(\frac{100}{3}\right) = \underline{\underline{(3.51)kT}}$$

for $E-E_f < -4kT \Longrightarrow f_{MB}^{app} = 1-\exp\left(\frac{E-E_f}{kT}\right)$

$$\Longrightarrow error = \left|\frac{f_{MB}^{app} - f_{FD}}{f_{FD}}\right| = +\exp\left(+2\frac{(E-E_f)}{kT}\right)$$

$$< \frac{3}{100} \Longrightarrow E-E_f < -\frac{kT}{2}\ln\left(\frac{100}{3}\right) = \underline{\underline{-1.75kT}}$$

Probability of being empty = $1 - f_{FD}$

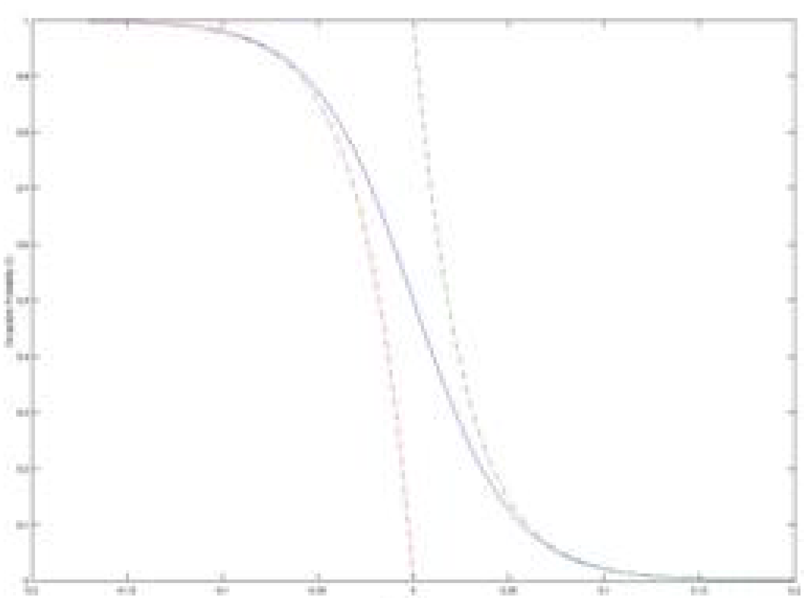
$$\overset{app.}{=} f_{MB} = \begin{cases} 1 - \exp\left(-\frac{E-E_f}{kT}\right) & \text{for } E-E_f > 4kT \quad ① \\ \exp\left(\frac{E-E_f}{kT}\right) & \text{for } E-E_f < -4kT \quad ② \end{cases}$$

$$① \Longrightarrow \text{error} = \left| \frac{f_{MB}^{\text{approximation}} - (1 - f_{FD})}{1-f_{FD}} \right| = \exp\left(-\frac{2(E-E_f)}{kT}\right) < \frac{3}{100}$$

$$\Longrightarrow E - E_f > \underline{\underline{1.75\,kT}}$$

$$② \Longrightarrow \text{error} = \exp\left(\frac{E-E_f}{kT}\right) < \frac{3}{100}$$

$$\Longrightarrow E - E_f < -kT\ln\left(\frac{100}{3}\right) = \underline{\underline{-3.51\,kT}}$$



2(a) – Metal

If the crystal has N atoms per unit vol.,

the outmost band has 2N States, ie. 2electrons

per atom (spin up and spin down). This band

has only one electron per atom ⟹ partially full

⟹ metal.

2(b) Here carrier (electron) Concentration $n = N$.

The out most band of a metal is similar to the

conduction band.

$$n = \int_{E_0}^{\infty} g_c(E)\, f_{FD}(E)\, dE$$

At $0^\circ K$. $f_{FD}(E) = 1$ for $E < E_f$

$= 0$ for $E > E_f$

therefore,

$$N = \int_{E_0}^{E_f} g_c(E) * 1\, dE$$

$$= \int_{E_0}^{E_f} \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_0)^{\frac{1}{2}} \, dE$$

$$N = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \left(\frac{2}{3} (E_f - E_0)^{3/2} \right)$$

$$\Longrightarrow E_f - E_0 = \left(\frac{3N}{8\pi} \right)^{\frac{2}{3}} \frac{h^2}{2m_e^*}$$

3 (a) @ 300°k, for Germanium:

$n_i = 2.4 \times 10^{13}\ cm^{-3}$, $N_C = 1.0 \times 10^{19}\ cm^{-3}$, $N_V = 5.4 \times 10^{18}\ cm^{-3}$

Phospherus is a group V material and therefore a donor atom.

Given that it's a shallow dopant, we can assume full ionization:

i.e. $N_d = 5 \times 10^{16}\ cm^{-3} \Longrightarrow n = N_d^+ + P \simeq N_d + P$

and because that $5 \times 10^{16} = N_d^+ >> n_i \Longrightarrow$

$$\boxed{n \simeq N_d^+ = 5 \times 10^{16}\ cm^{-3}}$$

@ equilibrium $np = n_i^2 \Longrightarrow P = \frac{(2.4 \times 10^{13})^2}{5 \times 10^{16}}\ cm^{-3}$

$$\boxed{P = 1.152 \times 10^{10}\ cm^{-3}}$$

(b) $E_f = E_C - KT \ln\left(\frac{N_C}{n}\right) = E_C - KT \ln\left(\frac{10^{19}}{5 \times 10^{16}}\right)$

$= E_C - 5.3\ KT = \boxed{E_C - 0.14\ eV}$

3(b) Following the same argument as a).

$N_A = 10^{18}\,cm^{-3} \simeq N_A^-$ $\quad N_d = 5\times10^{17}\,cm^{-3} \simeq N_d^+$

$p \simeq N_A^- - N_d^+ = \underline{\underline{5\times10^{17}}}\,cm^{-3}$

$n = \frac{n_i^2}{p} = \underline{\underline{1.152\times10^{9}\,cm^{-3}}}$

$E_F = E_V + KT \ln\left(\frac{N_V}{p}\right) =$

$= E_V + 0.026\,eV \ln\left(\frac{5.4\times10^{18}}{5\times10^{17}}\right) = \boxed{E_V + 0.062\,eV}$

4 (a):

1- Charge Neutrality $\Longrightarrow$ $\boxed{p + N_d^+ = n + N_A^-}$

2- Equilibrium mass action $\Longrightarrow$ $\boxed{np = n_i^2}$

$$\Longrightarrow \frac{n_i^2}{n} - n + (N_d^+ - N_a^-) = 0$$

$$\Longrightarrow n^2 - (N_d^+ - N_a^-)n - n_i^2 = 0$$

$$n = \left(\frac{N_d^+ - N_a^-}{2}\right) + \frac{\sqrt{(N_d^+ - N_a^-)^2 + 4n_i^2}}{2} \quad (n\text{-type})$$

$$E_f - E_{f_i} = KT\ln\left(\frac{n}{n_i}\right) = KT\ln\left(\frac{N_d^+ - N_a^-}{2n_i} + \sqrt{\left[\frac{N_d^+ - N_a^-}{2n_i}\right]^2 + 1}\right)$$

If we have full ionization, then, $\left(N_d^+ - N_a^- = N_d - N_a\right)$ by which we can rewrite the equations as functions of $N_d - N_a$.

4 (b)

$N_d = 10^{18}\,cm^{-3}$ (As), @ $750°C = 1023°K$, $\boxed{n_i \simeq 5\times10^{18}\,cm^{-3}}$

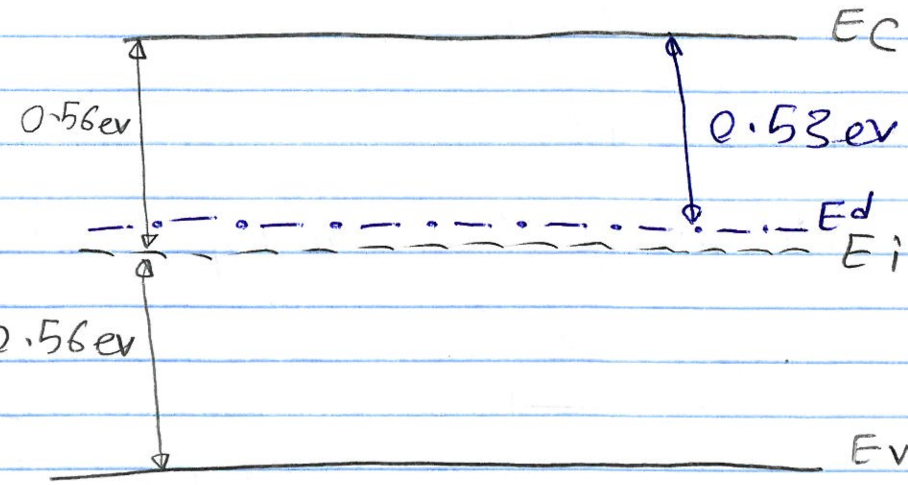
$$\Rightarrow n = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2} = \underline{\underline{5.52\times10^{18}}}\,cm^{-3}$$

$$P = \frac{n_i^2}{n} = \underline{\underline{4.52\times10^{18}}}\,cm^{-3}$$

$$E_f - E_{fi} = KT\ln\left(\frac{n}{n_i}\right) = KT\ln\left(\frac{5.52\times10^{18}}{5\times10^{18}}\right) = 0.1\times KT$$

$$= 0.1\times1023\times8.62\times10^{-5}\,eV = \underline{\underline{8.72\,meV}}$$

5(a)



By looking at the values, this question is about "deep donor". That means we can not assume full ionization (P.28 of note)

- Let's assume that this is an n-type material and the fermi level is close to Ec, so that $E_f > E_d + 3kT$ (This assumption seems reasonable from the plot on P.24 of the note. Even if a small portion (like $\frac{1}{1000}$) of the N_d^{deep} was ionized, the N_d^+ would be on the order of $10^{14}\,cm^{-3}$ which sets the fermi level at 0.3 eV above Ei)

$$\Longrightarrow N_d^+ = N_d \exp\left(-\frac{(E_f - E_d)}{kT}\right)$$

- Charge Neutrality:

$n = p + N_d^+$

- and $n >> p$ (for n-type material)

$$N_c \exp\left(\frac{E_f - E_c}{kT}\right) = N_d \exp\left(\frac{E_d - E_f}{kT}\right)$$

$$\frac{N_c}{N_d} = \exp\left(\frac{E_d - E_f - E_f + E_c}{kT}\right)$$

$$= \exp\left(\frac{E_d - 2E_f + E_c}{kT}\right)$$

$$\Rightarrow kT \ln\left(\frac{N_c}{N_d}\right) = E_d + E_c - 2E_f$$

$$= (E_c - 0.53) + E_c - 2E_f = 2(E_c - E_f) - 0.53$$

$$\Rightarrow E_c - E_f = \frac{kT}{2} \ln\left(\frac{N_c}{N_d}\right) + \frac{0.53}{2}$$

$$= \frac{kT}{2} \ln\left(\frac{2.8 \times 10^{19}}{10^{17}}\right) + \frac{0.53}{2} = \underline{\underline{0.37\,ev}}$$

$$\Rightarrow E_f - E_d = (0.53 - 0.37)\,ev = 0.16\,ev > 3kT$$

$\Rightarrow$ The assumption is correct!

$$n \simeq N_d^+ = N_d \exp\left(-\frac{(E_f - E_d)}{kT}\right) = \boxed{2.13 \times 10^{14}\ cm^{-3}}$$

5(a)

$$P = \frac{n_i^2}{n} = 0.47\times10^{6}\ cm^{-3} = \underline{\underline{470\times10^{3}\ cm^{-3}}}$$

5(b)

Using Neutrality again:

$$n = p + N_{As}^{+} + N_{Mn}^{+} \simeq N_{As}^{+} + N_{Mn}^{+}$$

$$\simeq N_{As}^{+} = 2\times10^{17}\ cm^{-3}$$

$$P = \frac{n_i^2}{n} = 500\ cm^{-3}$$

⊛ N_{Mn}^{+} is negligible: In this Case the fermi level is further from E_d, so N_{Mn}^{+} is less than of Part(a)

$$E_F = E_C - kT\ln\left(\frac{N_C}{n}\right) = E_C - 0.026\ln\left(\frac{2.8\times10^{19}}{2\times10^{17}}\right)$$

$$= \underline{\underline{E_C - 0.13\,ev}}$$

5(c) There are high concentration of ionized Boron acceptor atoms, so E_F will be well below E_d so that donors are fully ionized (assumption)

$\Longrightarrow$ $\underbrace{\text{Shallow Acceptor}}$

5(c) $N_d^+ = 10^{17}\ cm^{-3}$ $N_A^- = 2\times10^{17}\ cm^{-3}$

$\Longrightarrow P = N_A^- - N_d^+ = \underline{\underline{10^{17}\ cm^{-3}}}$

$n = \frac{10^{20}}{10^{17}} cm^{-3} = \underline{\underline{10^3\ cm^{-3}}}$

$E_f = E_v + KT \ln\left(\frac{N_v}{P}\right) = E_v + 0.025 \ln\left(\frac{2.5\times10^{19}}{10^{17}}\right)$

$= \underline{\underline{E_v + 0.14\,eV}}$

* Check Nd ionization:

$N_d^+ = N_d\left(1 - \exp\left(-\frac{E_d - E_f}{KT}\right)\right)$

$E_d - E_f = (0.56 - 0.53) + (0.56 - 0.14)\ eV$

$= \underline{\underline{0.45\,eV > 4KT}}$

$\Longrightarrow N_d^+ = N_d\left(1 - \exp\left(-\frac{0.45}{0.025}\right)\right) \simeq N_d$

$\Longrightarrow$ The assumption is OK!