Homework #8 Solutions

1(a) MOS Transistor : Bulk: $N_a = 10^{18}\ cm^{-3}$

$x_{ox} = 4nm = 4 \times 10^{-7} cm$

$W = 2L = 10^{-4} cm = 1\mu m$

Ignore Oxide Charges

Find V_{th} @ $V_{SB} = 0$:

$$V_{th} = \varphi_{ms} - 2\varphi_F - \frac{Q_B'}{C_{ox}'}$$

$$C_{ox}' = \frac{K_{ox}\epsilon_o}{x_{ox}} = \frac{(3.9)(8.85\times10^{-14}\ Fcm^{-1})}{4\times10^{-7}\ cm} = \underline{\underline{8.6\times10^{-7}\ Fcm^{-2}}}$$

$$\varphi_F = -\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) = -0.0256\ln\left(\frac{10^{18}}{10^{10}}\right)v = \underline{\underline{-0.4721}}$$

$$Q_B' = -\sqrt{2K_s\epsilon_o q N_a |2\varphi_F|} = \underline{\underline{-5.59\times10^{-7}\ Ccm^{-2}}}$$

$$\varphi_{ms} = \chi_s - \left(\chi_s + \frac{E_g}{2} + |\varphi_F|\right) = -\left(\frac{E_g}{2} + |\varphi_F|\right) = \underline{\underline{-1.03}}$$

$$V_{th} = -1.03 + 0.944 + \frac{5.59\times10^{-7}\ Ccm^{-2}}{8.6\times10^{-7}\ Fcm^{-2}} = \underline{\underline{0.564v}}$$

1b) To use the equation given, we need to find Q_d^{max}

$$Q_d^{max} = -\sqrt{2K_s\epsilon_o q N_a |V_{CB} - 2\varphi_F|}$$

$$C_S^{depl} = \frac{d Q_d^{max}}{d V_{CB}} = \frac{1}{2}\sqrt{\frac{2 k_S \epsilon_0 q N_a}{|V_{CB} - 2\varphi_F|}}\Bigg|_{V_{CB}=0}$$

$$\alpha = \frac{C_S^{depl}}{C_{ox}'} = \frac{1}{2C_{ox}'}\left(\frac{2 k_S \epsilon_0 q N_a}{2\varphi_F}\right)^{\frac{1}{2}} = \underline{0.34}$$

1d) i) $V_S = 0$ $\quad V_B = 0V$ $\quad V_G = 3V$ $\quad V_D = 1V$

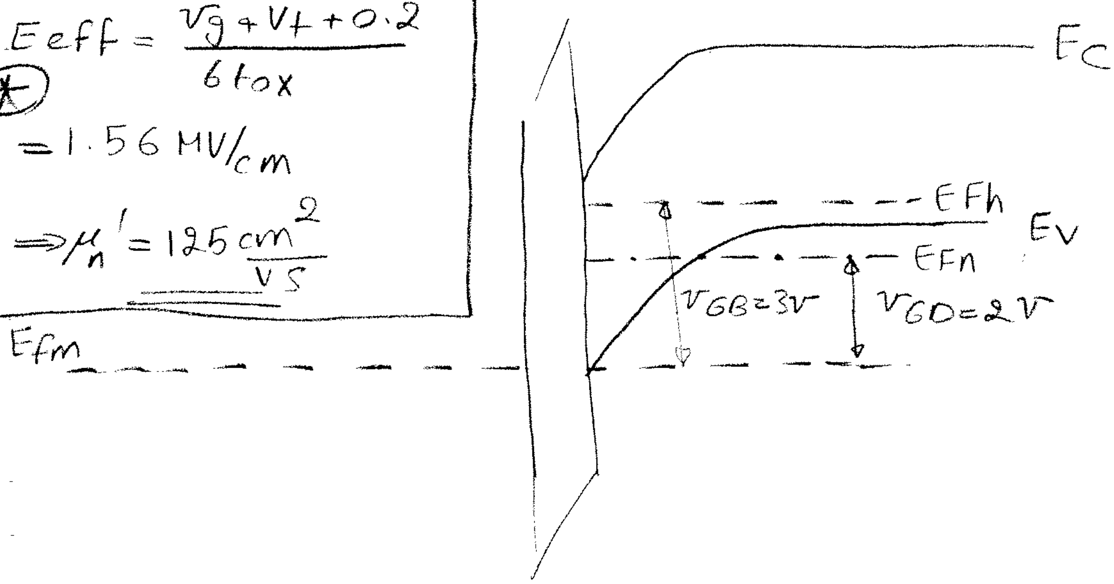
$V_G > V_{th} \Rightarrow$ Device is <u>ON</u>

$$V_{DS,sat} = \frac{V_{GS} - V_{th}}{1+\alpha} = \frac{3 - 0.564}{1 + 0.34} = 1.82V$$

$V_{DS} < V_{DS,sat} \Rightarrow$ Device is in <u>linear region</u>

$$I_D = \frac{W}{L}\mu_n' C_{ox}'\left[(V_{GS} - V_{th})V_{DS} - \frac{1}{2}(1+\alpha)V_{DS}^2\right] = \underline{0.380\ mA}$$

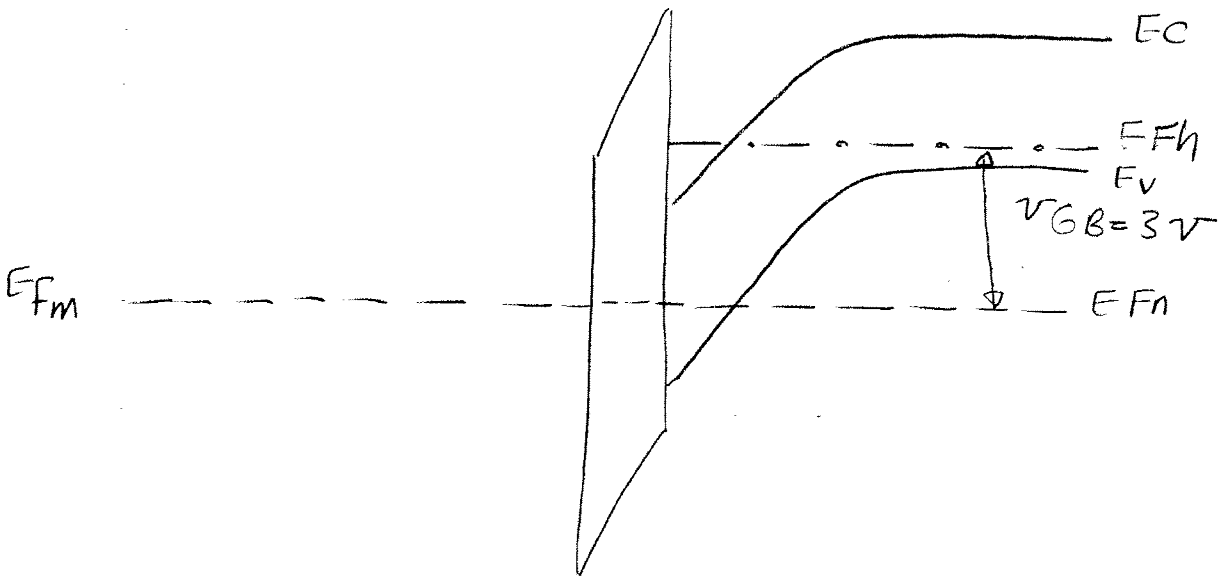
$$\boxed{E_{eff} = \frac{V_g + V_t + 0.2}{6 t_{ox}} = 1.56\ MV/cm \Rightarrow \mu_n' = 125\ \frac{cm^2}{Vs}}$$



1d) ii) $V_S = 0V$, $V_B = 0V$, $V_G = 3V$, $V_D = 3V$

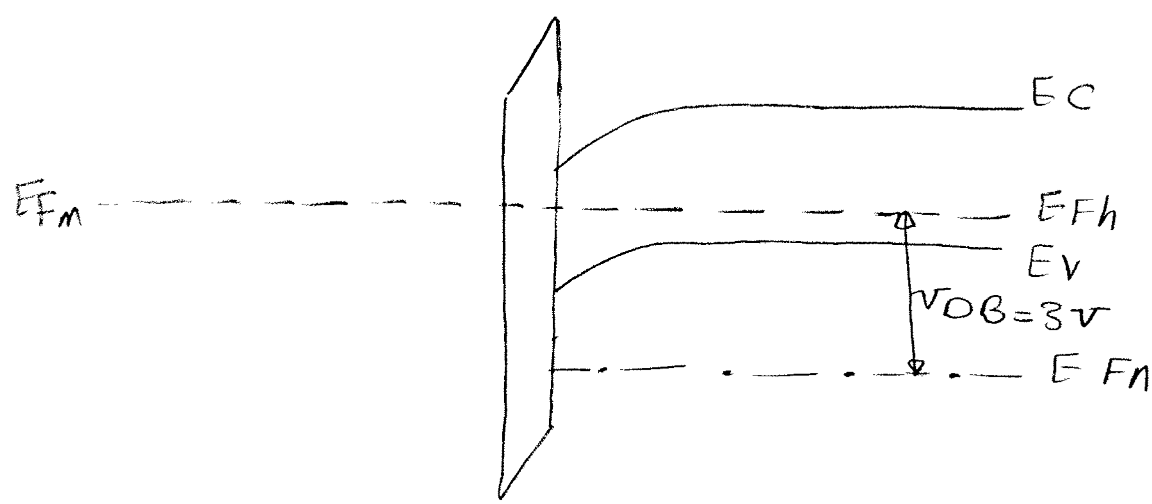
Device is <u>ON</u> and is in the <u>Saturation</u> region

$$I_D = \frac{W}{L}\,\mu_n'\,C_{ox}'\left[\frac{(V_{GS}-V_{th})^2}{2(1+\alpha)}\right] = \underline{\underline{0.476\ mA}}$$



1d) iii) $V_S=0$, $V_B=0V$, $V_G=0V$ $V_D=3V$

$V_G < V_{th} \Rightarrow$ Device is <u>OFF</u> $\Rightarrow I_D=0$



2 a) MOS P-channel, $V_{th} = -0.4V$ $S/B = 0V$

$V_G = -3V$ $V_D = -2V$ $X_{ox} = 10 \times 10^{-7} cm = 10^{-6} cm$

$N_d = 10^{17} cm^{-3}$ $W/L = 2$

Find V_{FB}:

$$V_{th} = V_{FB} - 2\varphi_F - \frac{Q_B'}{C_{ox}'} \quad ①$$

$$\varphi_F = \frac{kT}{q} \ln\left(\frac{N_d}{n_i}\right) = 0.0256V \ln\left(\frac{10^{17}}{10^{10}}\right) = \underline{\underline{0.41V}}$$

$$Q_B' = \sqrt{2k_s\epsilon_0 q N_d |2\varphi_F|} = \underline{\underline{1.65 \times 10^{-7}}} C\, cm^{-2}$$

$$C_{ox}' = \frac{(3.9)(8.85 \times 10^{-14} F cm^{-1})}{10^{-6} cm^{-1}} = \underline{\underline{3.45 \times 10^{-7} F cm^{-2}}}$$

$$① \Longrightarrow -0.4V = V_{FB} - 2(0.41)V - \frac{(1.67 \times 10^{-7} C cm^{-2})}{3.45 \times 10^{-7} F cm^{-2}}$$

$$\Longrightarrow V_{FB} = \underline{\underline{0.9V}}$$

2b) PMOS $\Rightarrow$ $V_g < V_{th}$ $\Longrightarrow$ Device is $\underline{\underline{\text{ON}}}$

$$\alpha = \frac{1}{2C_{ox}}\left(\frac{2k_s\epsilon_0 q N_d}{2\varphi_F}\right)^{\frac{1}{2}} = \underline{\underline{0.29}}$$

$$V_{DS,sat} = \frac{V_G - V_{th}}{1+\alpha} = \underline{\underline{-2.02V}}$$

$V_{DS} > V_{DS,sat}$ $\Longrightarrow$ $\underline{\underline{\text{Linear Region}}}$

2c) $I_D = \frac{W}{L}\mu_p' C_{ox}' \left[(V_{GS} - V_{th}) V_{DS} - \frac{1}{2}(1+\alpha) V_{DS}^2 \right]$

$E_{eff} = \frac{V_g + V_t - 2.3V}{6\, X_{ox}} = -0.95\ MVcm^{-1} \Rightarrow \mu_p' \simeq 50 \frac{cm^2}{VS}$

$\Rightarrow I_D = \underline{0.0897\,mA}$

2d)

$V_{th} = V_{FB} - 2\varphi_F - \frac{\sqrt{2 k_S \epsilon_0 q N_d \,|V_{SB} - 2\varphi_F|}}{C_{ox}'}$

$V_{DS} = V_{DS,sat} = \frac{V_{GS} - V_{th}}{1+\alpha}$

$\Rightarrow (1.29)(-2) = -3 - V_{th}$

$\Rightarrow V_{th} = -3 + 2 \times 1.29 = -0.42V$

$\Rightarrow -0.42 = 0.9 - 0.82 - (|V_{SB} - 0.82|)^{\frac{1}{2}}(0.53)$

$\Rightarrow V_{SB} - 0.82 = -0.88 \Rightarrow V_{BS} = 0.07V$

And the device will go to the Saturation mode.

2e) $V_{SB} = 0V \quad N' = 10^{12}\,cm^{-2}$

$\Delta V_{th} = \frac{-qN'}{C_{ox}'} = -\frac{(1.6\times10^{-19}c)(10^{12}cm^{-2})}{3.95\times10^{-7}Fcm^{-2}} = \underline{-0.46V}$

$V_{th,new} = -0.4 - 0.46V = \underline{-0.86V}$

$V_{DS,sat} = \frac{V_{GS} - V_{th,new}}{1+\alpha} = \underline{-1.66V}$

$\Rightarrow$ Device is ON and is in Saturat

3) $L = 0.1\mu m = 10^{-5}$ cm $\quad V_{DS} = V_{DS,sat} + 0.4\,V$

$I_D = 10^{-6}$ A $\quad N_a = 2\times10^{18}$ cm^{-3}

$\Longrightarrow$ Device is in Saturation :

Equation 16 from the note. $\Longrightarrow$ $\Delta L \simeq \sqrt{\dfrac{2k_s\varepsilon_0}{qN_a}|V_{DS}-V_{DS,sat}|} = \underline{1.62\times10^{-6}\text{ cm}}$

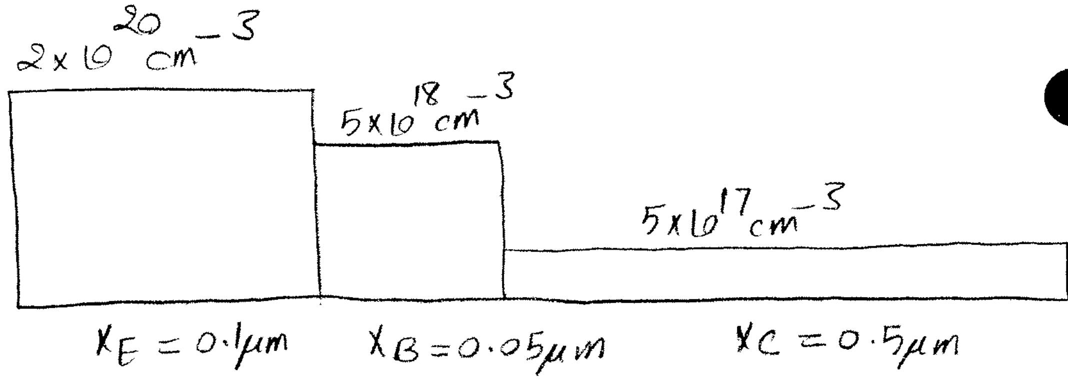
$$I_D' = \left(\frac{L}{L-\Delta L}\right) I_D = \left(\frac{10^{-5}}{10^{-5}-1.62\times10^{-6}}\right)\times10^{-6}\,A = 1.19\times10^{-6}\,A$$

$$g_{d,sat} = \frac{1}{R_{out}} = \frac{I_D'-I_D}{\Delta V_{DS}} = \frac{0.19\,\mu A}{0.4\,V} = 4.75\times10^{-7}\,\mho$$

$\Longrightarrow R_{out} = \underline{2.11\ M\Omega}$

It is also possible to calculate the output resistance in saturation (R_{out} or r_{ds}) by differentiating for I_{DS}' above with respect to V_{DS} (via dependence of ΔL on V_{DS}). The result is the small signal output resistance about the operating point specified ($V_{DS}-V_{DS,sat}=0.4\,V$) rather than the average output resistance in saturation over the range of $0 < V_{DS}-V_{DS,sat} < 0.4$ as calculated above.

4a)



$\alpha_F = \frac{I_C}{I_E} = \alpha_T \gamma \delta$ ← δ is 1 because there is no recombination

$D_{PE} = 1\ cm^2/s \quad D_{nB} = 5\ cm^2/s \quad D_{PC} = 6.5\ cm^2/s$

$L_{nB} = \sqrt{D_{nB}\tau_{nB}} = \sqrt{5(0.5\mu s)} = \underline{\underline{16\mu m}}$

$\alpha_T \simeq 1 - \frac{X_B^2}{2(L_{nB})^2} = 1 - \frac{(0.05\times10^{-4})^2}{(16\times10^{-4})^2}$ ← True because $X_B \ll L_{nB}$

$= 0.99999 \simeq 1 \implies \alpha_F \simeq \gamma_F$

$$\alpha_F = \gamma_F = \frac{1}{1+\frac{X_B}{X_E}\frac{N_{AB}}{N_{dE}}\frac{D_{PE}}{D_{NB}}} = \frac{1}{1+\frac{0.05\mu m}{0.1\mu m}\frac{5\times10^{18}}{2\times10^{20}}\times\frac{1}{5}}$$

$= \underline{\underline{0.9925}}$

$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \underline{\underline{400}}$

4a) from equation (38):

$$a_{12} = \alpha_F I_{ES} \Longrightarrow \frac{q A D_{nB} n_i^2}{X_B N_{aB}} = \alpha_F I_{ES}$$

$$\Longrightarrow J_{ES} = \frac{q D_{nB} n_i^2}{X_B \alpha_F N_{aB}} = \frac{(1.6\times10^{-19}C)(5cm^2/s)(10^{20}cm^{-6})}{(0.05\times10^{-4}cm)(0.9925)(5\times10^{18}cm^{-3})}$$

$$= \underline{3.22\times10^{-12}\,A\,cm^{-2}}$$

4b) $\alpha_R = \alpha_T \gamma_R \stackrel{\sim}{=} \gamma_R$

$$\gamma_R = \frac{1}{1+\frac{X_B}{X_C}\frac{D_{pC}}{D_{nB}}\frac{N_{aB}}{N_{dC}}} = \frac{1}{1+\frac{(0.05\mu m)}{(0.5\mu m)}\times\frac{6.5}{5}\times\frac{5\times10^{18}}{5\times10^{17}}}$$

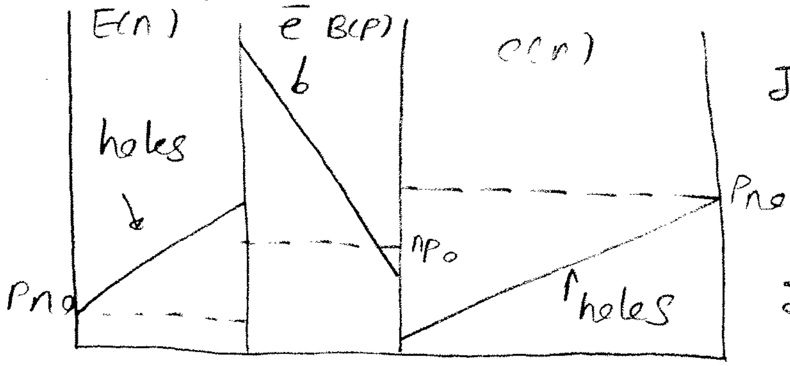
$$= \underline{0.435}$$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R} = 0.769$$

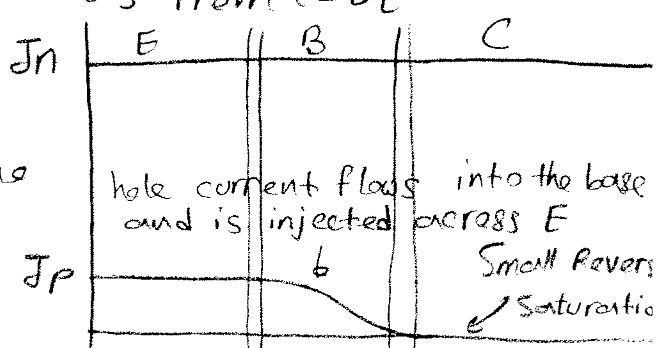
$$a_{12} = \alpha_R I_{CS} \Longrightarrow I_{CS} = \underline{7.35\times10^{-12}\,A\,cm^{-2}}$$

4c) For (a), forward active.

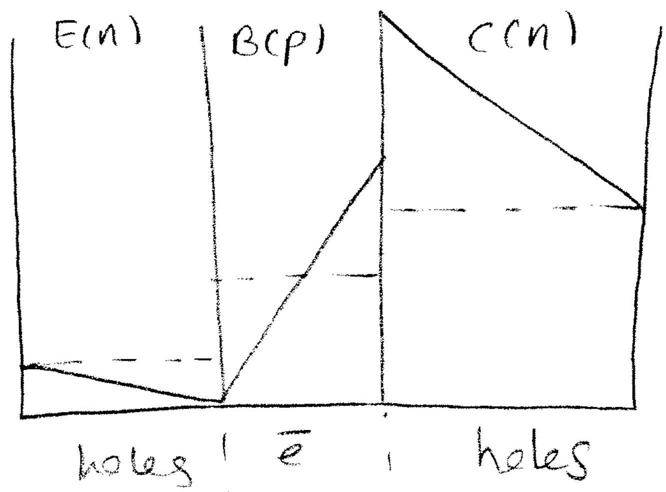
Minority Charge Densities



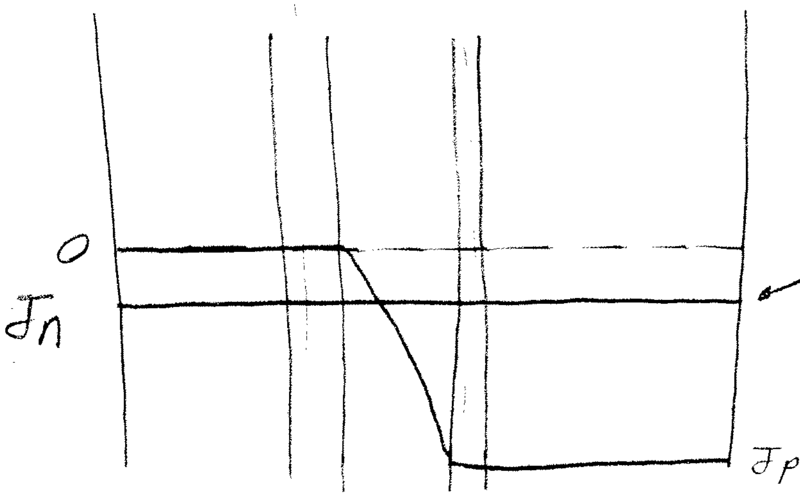
Positive Current direction defined as from C → E



For (b) reverse active:



Minority Charge Density



J_n is now negative

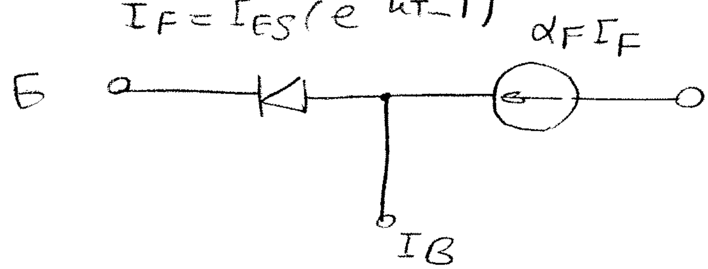
- J positive direction is C → E
- $J_{pc} > J_n$ since $\alpha_R \ll 1$
- $J_n \cong$ Constant ∵ $\alpha_T \cong 1$

4d) $A_E = A_C = 10^{-5}\ cm^2$

$I_B = 1\mu A \qquad V_{CE} = 3V$

Assume that we are in the forward active mode ($V_{BE} > 0 \quad V_{BC} < 0$)

$I_F = I_{ES}(e^{\frac{qV_{BE}}{kT}} - 1)$



(Ignoring the reverse saturation current)

$\Longrightarrow \alpha_F I_F + I_B = I_F$

$\Longrightarrow (0.9925) I_F + 1\mu A = I_F$

$\Longrightarrow 7.5 \times 10^{-3} \times I_F = 1\mu A \Longrightarrow I_F = 1.33 \times 10^{-4} A$

$\Longrightarrow I_C = \alpha_F I_F = \underline{0.132 mA}$

Checking the assumption:

$I_{ES} = J_{ES} \times A_E = 3.22 \times 10^{-12} \times 10^{-5} = 3.22 \times 10^{-17}$

$\Longrightarrow I_F = 1.33 \times 10^{-4} = 3.22 \times 10^{-17} (e^{\frac{qV_{BE}}{kT}})$

$\Longrightarrow V_{BE} = \underline{0.74 V}$ and $V_{BC} = \underline{-2.26 V}$ OK ✓