

Movement of Free Carriers in Semiconductors

A. Thermal Motion

In thermal equilibrium, mobile (conduction band) electrons will be in random thermal motion. According to statistical mechanics, the average kinetic energy of a particle is $(1/2)kT$ for each degree of freedom. Therefore since there are three dimensions:

$$\frac{1}{2}m_e^*v_{th}^2 = \frac{3}{2}kT \quad (1)$$

$$v_{th} \sim 10^7 \text{ cm/s @ } 300^\circ\text{K}$$

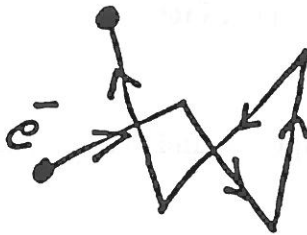
Lattice atoms also vibrate. Due to quantum mechanical considerations, discrete allowed vibrational states exist which can be modeled as particles called phonons.

electron-atom collisions \equiv phonon (lattice) scattering

In a perfect lattice at 0°K there is no scattering since the effect of the lattice has already been included in the effective mass.

Other scattering mechanisms are:

- Ionized impurity scattering (important at high doping concentrations).
- Neutral impurity scattering (usually negligible).
- Electron-electron and electron-hole scattering (important at high carrier concentration).
- Crystal defects (can be minimized by using high quality crystals)



In thermal equilibrium, thermal motion is random, resulting in zero net current.

The average distance between collisions (l_n) is defined as the mean free path, $l_n = 100 \text{ \AA} - 1 \mu\text{m}$ in silicon.

Since $v_{th} \sim 10^7 \text{ cm/sec}$, the mean free time between collisions is:

$$\tau_n \sim \frac{10^{-5} \text{ cm}}{10^7 \text{ cm/s}} \sim 1 \text{ ps}$$

B. Drift (Field-Driven Motion)

Now apply an electric field across the crystal. Motion of electron is in the direction opposite to electric field on the average. This process is called **drift**.

The drift velocity, v_d , is the average velocity in the field direction. In response to an electric field,

impulse (force \times time) = momentum gained

$$(-q\mathcal{E})\tau_n = m_e^* v_d$$

$$v_d = -\frac{q\tau_n}{m_e^*} \mathcal{E} = -\mu_n \mathcal{E} \quad (2)$$

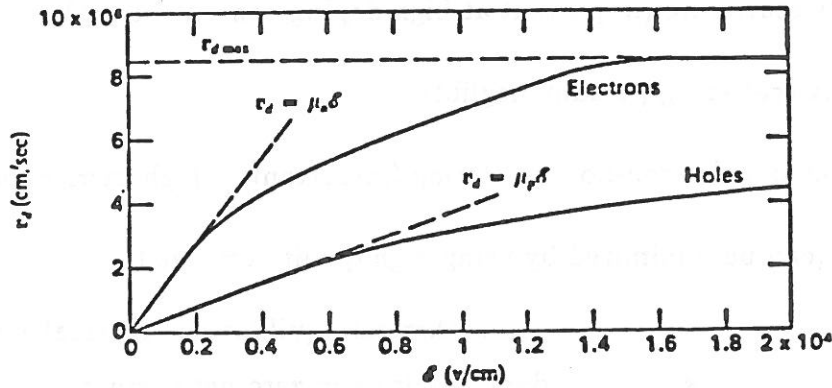
where

$$\mu_n = \frac{q\tau_n}{m_e^*}$$

is the electron mobility and determines how strongly an electron is influenced by an electric field. Similarly for holes,

$$v_d = \frac{q\tau_p}{m_h^*} \mathcal{E} = \mu_p \mathcal{E}. \quad (3)$$

The predicted linear relationship between v_d and \mathcal{E} holds only for small \mathcal{E} .



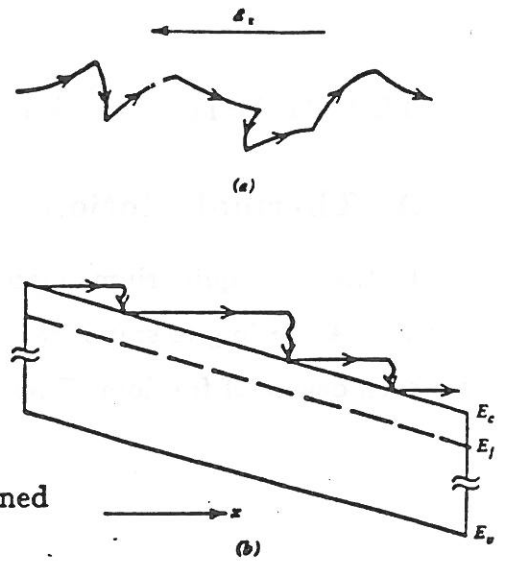
At high fields, v_d becomes comparable to v_{th} , which is an upper bound on v_d , and v_d saturates.

$$v_{sat} \equiv \text{scattering limited velocity} \cong \begin{cases} 10^7, & \text{cm/sec for } e^- \text{ in Si.} \\ 6 \times 10^7, & \text{cm/sec for } h^+ \text{ in Si.} \end{cases} \quad (4)$$

The critical field at which this occurs is

$$\mathcal{E}_{crit} \sim 1.5 \times 10^4 \text{ V/cm} = 1.5 \text{ V}/\mu\text{m}.$$

This is an important physical limit for very small semiconductor devices (MOS in particular).



Scattering processes act in parallel.

probability of scattering = $\frac{dt}{\tau_i}$ for each process

$$\frac{dt}{\tau} = \sum_i \frac{dt}{\tau_i} \quad (5)$$

$$\frac{1}{\mu} = \sum_i \frac{1}{\mu_i} \quad (6)$$

Transport is limited by the process that produces the shortest scattering time or lowest mobility.

Lattice (phonon) scattering dominates at low doping ($\mu = \text{const}$). Ionized impurity scattering dominates for high doping reducing the mobility as N_d and N_a increase. Note that μ_n is greater in Ge and GaAs than in Si.

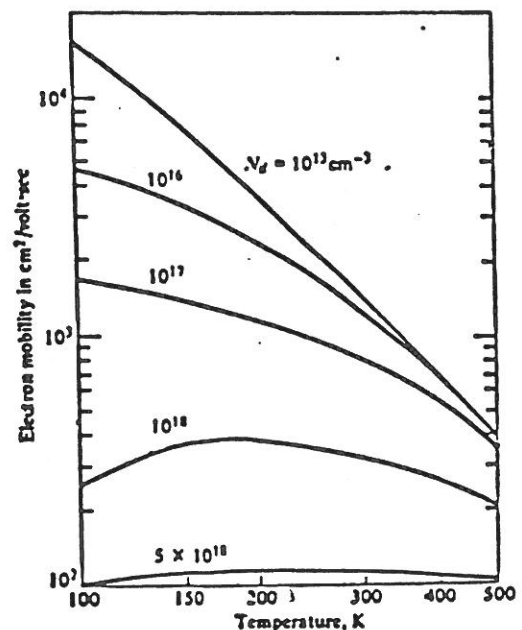
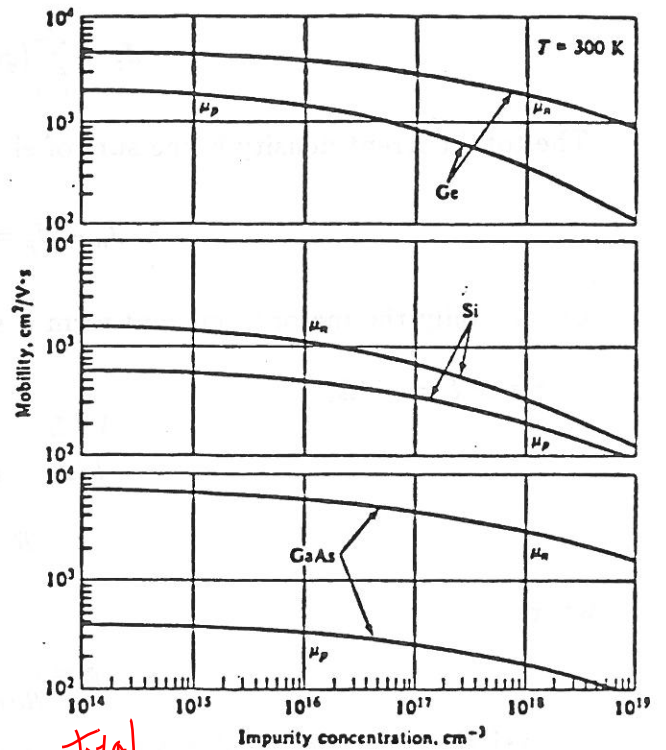
As temperature rises, lattice vibrations increase, increasing the amount of lattice scattering and reducing the mobility,

$$\mu_{\text{lattice}} \propto T^{-n}, \quad n \sim 2.5$$

In contrast, impurity scattering is reduced at high temperature since carriers move faster and remain near impurities for shorter times.

$$\mu_{\text{impurity}} \propto T^n, \quad n \sim 1.5$$

Therefore, at higher temperatures lattice scattering tends to dominate over impurity scattering.



Drift Current

The drift of electrons under an applied electric field results in a current flow. The current density of electron current is

$$\frac{J_n}{A} = J_n = \sum_{i=1}^n (-qv_i) = -nqv_d = nq\mu_n \mathcal{E}.$$

Holes move in opposite direction but have opposite charge.

$$J_p = \sum_{i=1}^p (qv_i) = pqv_d = pq\mu_p \mathcal{E}.$$

The total current density is the sum of electron and hole current densities,

$$J = J_n + J_p = \frac{I}{A} = (nq\mu_n + pq\mu_p) \mathcal{E} \quad (7)$$

Usually only the majority current term is significant.

Since $\mathcal{E} = V/L$,

$$\frac{V/L}{I/A} = \frac{1}{q\mu_n n + q\mu_p p} = \rho \quad (8)$$

$$R = \rho \frac{L}{A} = \frac{V}{I}$$

where

$$\rho = \frac{1}{q\mu_n n + q\mu_p p} = \frac{1}{\sigma}. \quad (9)$$

ρ is resistivity in $\Omega\text{-cm}$ and σ is conductivity in $(\Omega\text{-cm})^{-1}$.

Note: The mobility depends on the total dopant concentration.

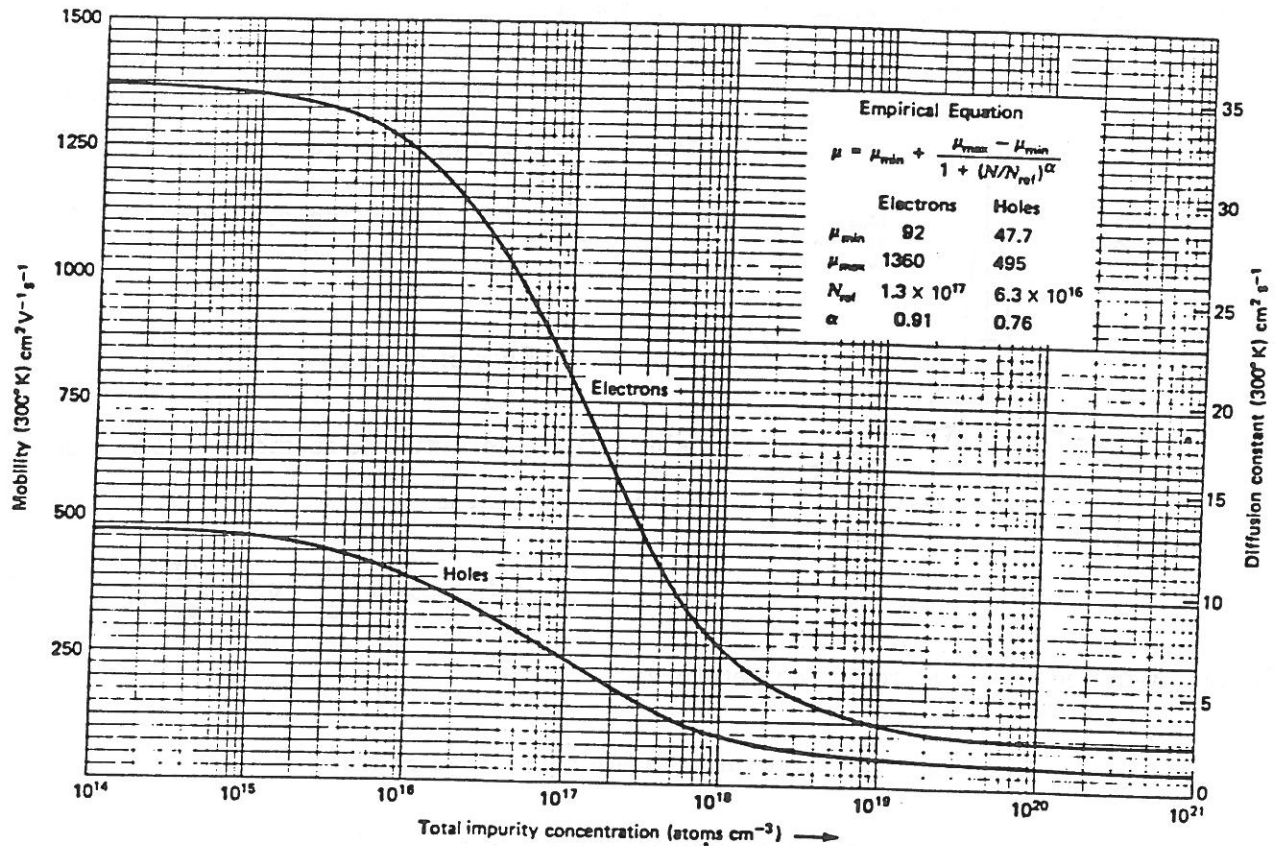


Figure 1.12 Electron and hole mobilities in silicon at 300°K as functions of the total dopant concentration. The values plotted are the results of curve fitting measurements from several sources. An empirical equation for the data is shown in the inset.

Note: This graph can only be used for **uncompensated** material (single dopant type). Compensated material will have a larger value of ρ for the same net doping concentration.

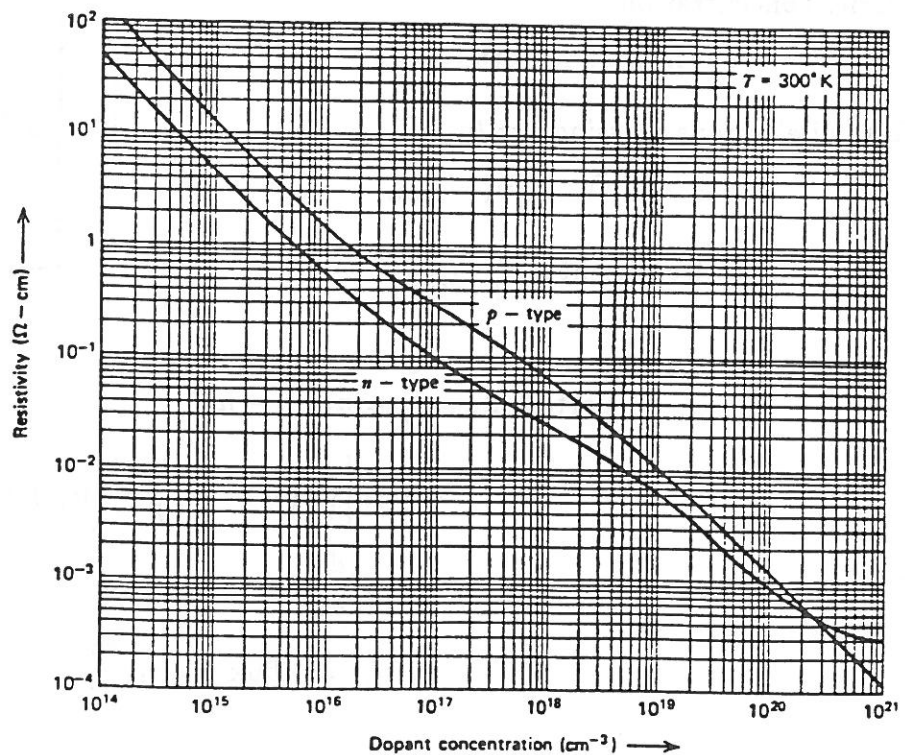


Figure 1.11 Resistivity as a function of dopant concentration at 300 °K for *n*- and *p*-type silicon.⁴

C. Diffusion (Gradient-Driven Motion)

Diffusion occurs from regions of high concentration towards region of low concentration.

The equation governing diffusion is Fick's first law:

$$\text{Flux of carriers} = \phi = -D \frac{dn}{dx} \quad (10)$$

where D is the diffusion coefficient. Therefore, the diffusion current is

$$I = qAD_n \frac{dn}{dx} - qAD_p \frac{dp}{dx} \quad (11)$$

In lightly-doped silicon at room temperature,

$$D_n = \text{the electron diffusion constant} \cong 38 \text{ cm}^2/\text{sec}$$

$$D_p = \text{the hole diffusion constant} \cong 13 \text{ cm}^2/\text{sec}$$

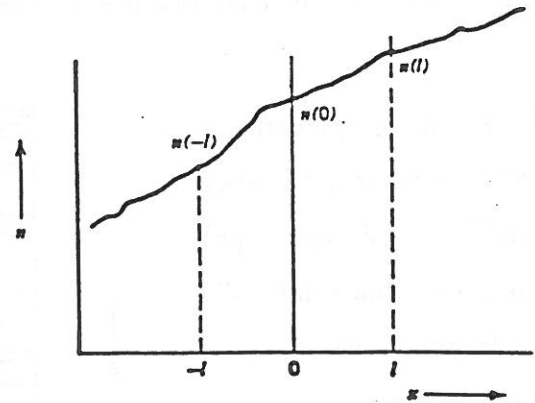
For one-dimensional diffusion where $n(x)$ is the carrier concentration

$$l = \text{mean free path} = v_{th} \tau_n.$$

Rate of electrons crossing $x = 0$:

$$\text{from left : } \frac{1}{2} n(-l) v_{th},$$

$$\text{from right : } \frac{1}{2} n(l) v_{th}.$$



$$\text{particle flux} = \frac{1}{2} v_{th} [n(-l) - n(l)]$$

$$= \frac{1}{2} v_{th} \left[\left(n(0) - \frac{dn}{dx} l \right) - \left(n(0) + \frac{dn}{dx} l \right) \right]$$

$$= -v_{th} l \frac{dn}{dx}$$

$$J_n = -q \times \text{flux} = ql v_{th} \frac{dn}{dx} \quad (12)$$

Based on the the equipartition of energy, due to statistical mechanics in 1D:

$$\frac{1}{2}m_e^*v_{th}^2 = \frac{1}{2}kT$$

Since $l = v_{th}\tau_n$,

$$J_n = q \left(\tau_n v_{th}^2 \right) \frac{dn}{dx} = q \left(\tau_n \frac{kT}{m_e^*} \right) \frac{dn}{dx} = q \left(\frac{kT}{q} \mu_n \right) \frac{dn}{dx}. \quad (13)$$

Since the definition of diffusion coefficient is given by Fick's law:

$$J_n = qD_n \frac{dn}{dx}, \quad (14)$$

$$D_n = \frac{kT}{q} \mu_n. \quad (15)$$

This is known as the Einstein Relation.

If both drift and diffusion are important,

$$I_n = qA \left(\mu_n n \mathcal{E} + D_n \frac{dn}{dx} \right) \quad (16)$$

$$I_p = qA \left(\mu_p p \mathcal{E} - D_p \frac{dp}{dx} \right) \quad (17)$$

These are the basic transport equations will use in deriving device characteristics.

Integrated Circuit Resistor

$$G = R^{-1} = \left(\rho \frac{L}{A} \right)^{-1} = \sigma \frac{A}{L}$$

$$\sigma = \frac{1}{\rho} = q\mu_n n + q\mu_p p$$

Examine the resistance of a diffused p -region.

Considering any thin layer,

$$dG(x) = q\mu_p p(x) \frac{W}{L} dx, \quad (W dx = A)$$

$$G = \frac{W}{L} \int_0^L q\mu_p p(x) dx$$

Most of the current is carried in the high conductance (high concentration) region. If $N_d \ll N_a$, then $p(x) \cong N_a(x)$. Recall that $\mu = f(N_d, N_a)$. Use μ for $N_a/2$.

$$G \cong \frac{W}{L} q\bar{\mu}_p \int_0^L p(x) dx \cong \frac{W}{L} q\bar{\mu}_p N'$$

where N' is the total doping concentration.

If we define

$$g \equiv N' q\bar{\mu}_p,$$

then g is the conductance of a square region ($W = L$).

$$R = G^{-1} = \frac{L}{W} \frac{1}{g} = \frac{L}{W} R_{\square} \quad (20)$$

R_{\square} is the sheet resistance given in units of Ω/\square . R_{\square} is determined by the process (doping profile). Different resistors are often wanted with the same process. This can be easily achieved by changing the aspect ratio (L/W) of the resistor.

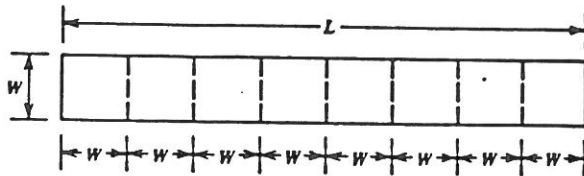
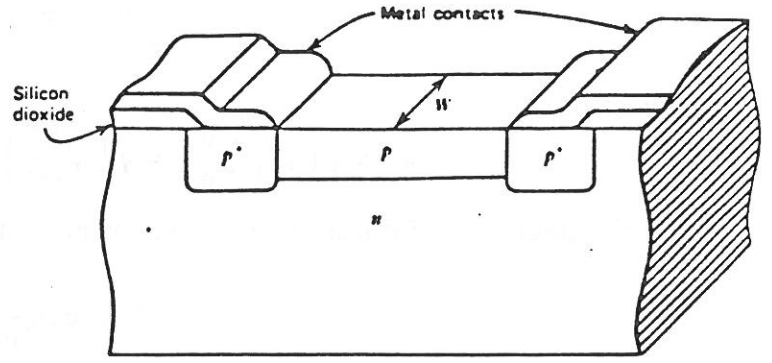
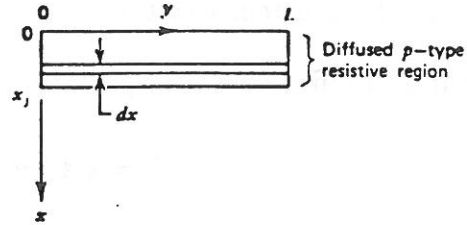


Figure 1.30 The number of squares describing the surface dimensions of a resistor is given by the ratio L/W .



(18)



(19)

Limitations to control of resistance ($\sim 30\%$):

- Patterning (lateral diffusion)
- Dose and profile (ion implantation produces better control)

Ratios, however, are much more consistent ($\sim 5\%$, $R_{\square} \cong \text{constant}$).

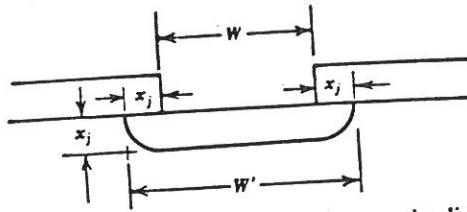


Figure 1.32 Lateral diffusion changes the dimensions of the resistor from the nominal mask dimensions.

For larger resistance values without excessive area can use:

- Epitaxial layer
- Pinch resistor (less repeatable)
- Deposited layer

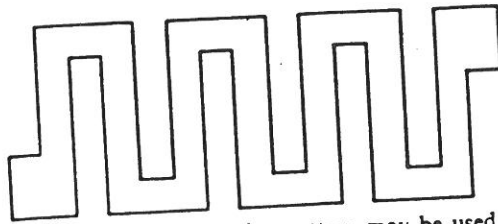


Figure 1.31 A serpentine pattern may be used when a long, high-valued resistor must be designed.