Name Solutions

Mean: 26.4
Median: 28
Std. Dev: 10.4

Exam #1 — EE 482

Winter 2010

The test is open book/open notes. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses (50 total).

1. The electron concentration in a region of silicon depends linearly on depth with concentration of $5 \times 10^{15}\,\text{cm}^{-3}$ at surface ($x = 0$) and $10^{15}\,\text{cm}^{-3}$ at depth of $x = 500\,\text{nm}$. If the vertical electron current density in this region is constant at $J_n = 100\,\text{A/cm}^2$, calculate the electric field near $x = 500\,\text{nm}$. You may assume that the mobility is constant at $1250\,\text{cm}^2/\text{Vs}$. (12)

$$J_n = q\left(\mu_n n \mathcal{E} + D_n \frac{dn}{dx}\right)$$

$$n(500\,\text{nm}) = 10^{15}\,\text{cm}^{-3}$$

$$\left.\frac{dn}{dx}\right|_{500\,\text{nm}} = \frac{10^{15} - 5\times10^{15}\,\text{cm}^{-3}}{5\times10^{-5}\,\text{cm}} = 8\times10^{19}\,\text{cm}^{-4}$$

$$\mu_n = 1250\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}} \qquad D_n = \frac{kT}{q}\mu_n$$

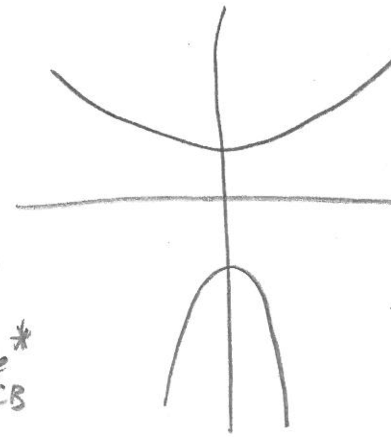
$$J_n = q\mu_n\left(n\mathcal{E} + \frac{kT}{q}\frac{dn}{dx}\right)$$

$$\mathcal{E} = \frac{1}{n}\left[\frac{J_n}{q\mu_n} - \frac{kT}{q}\frac{dn}{dx}\right] = \frac{1}{10^{15}\,\text{cm}^{-3}}\left[\frac{100\,\text{A/cm}^2}{(1.6\times10^{-19}\,\text{C})\left(1250\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}\right)} - (0.026\,\text{V})(8\times10^{19}\,\text{cm}^{-4})\right]$$

$$= 10^{-15}\,\text{cm}^3\left[\frac{100\,\text{V/cm}^4}{2\times10^{-16}} - 2.08\times10^{18}\,\frac{\text{V}}{\text{cm}^4}\right]$$

$$= 2.58\times10^{3}\,\frac{\text{V}}{\text{cm}} = 2580\,\frac{\text{V}}{\text{cm}}$$

2. (a) Sketch the band diagram (E vs. k along main symmetry directions) for a semiconductor which has $\mu_p > \mu_n$ and $N_v < N_c$. The qualitative relations should be clear from your sketch. Assume scattering lifetimes are equal for holes and electrons. Briefly note reasoning. (10)



i) $\mu_p = \frac{q\tau_p}{m_h^*}$, $\mu_n = \frac{q\tau_n}{m_e^*}$ $\quad \mu_p > \mu_n \Rightarrow \frac{1}{m_h^*} > \frac{1}{m_e^*}$

$\tau_p = \tau_n \quad \Rightarrow \left(\frac{d^2E}{dk^2}\right)^{VB} > \left(\frac{d^2E}{dk^2}\right)^{CB}$

ii) $N_v = 2N_{max}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$

$N_v < N_c \Rightarrow m_h^* < m_e^*$ and/or $N_{max}^{VB} < N_{min}^{CB}$

(b) Calculate the intrinsic carrier concentration at 300 K in a semiconductor which has a single valence band maxima given by $E = E_v - (\hbar^2 k^2/2m_0)$ and eight conduction band minima in the $\langle 111 \rangle$ directions with $E = E_c + 2\hbar^2(k - k_0)^2/m_0$ where $k_0 = (\pm a, \pm a, \pm a)$, $E_c - E_v = 1\,\text{eV}$ and m_0 is the free electron mass. (10)

$n_i = \sqrt{N_c N_v}\, \exp(-E_g/2kT) \qquad E_g = E_c - E_v = 1\text{eV}$

$N_c = 2\underbrace{N_{min}}_{8}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \qquad m_e^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)_{CB}} = \frac{\hbar^2}{4\hbar^2/m_0} = \frac{m_0}{4}$

$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \qquad m_h^* = m_0$

$n_i = 2\left(\frac{2\pi m_0 kT}{h^2}\right)^{3/2}\left[8\left(\frac{1}{4}\right)^{3/2}(1)^{3/2}\right]^{1/2} \exp\left(-\frac{1\text{eV}}{0.052\text{eV}}\right)$

$= 2\left(\frac{2\pi(9.11\times10^{-31}\cancel{kg})(0.026\cancel{eV})}{(6.63\times10^{-34}\cancel{kg}\,m^2/\cancel{s})(4.14\times10^{-15}\cancel{eV\cdot s})}\right)^{3/2} 4.45\times10^{-9} = 1.12\times10^{17}\,m^{-3}$

$= 1.12\times10^{11}\,cm^{-3}$

EE 482 Wild

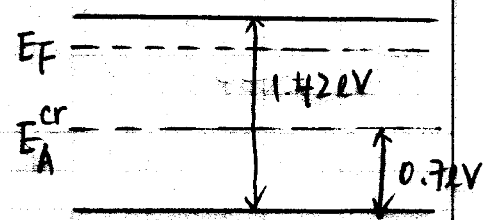
Exam 1.

3(A) $N_D = 10^{16} \text{cm}^{-3}$ $N_A = 2\times10^{15} \text{cm}^{-3}$ → assume all shallow dopants → fully ionized.

assume that this is an n-type material

Steps:
- Proof of assumptions
- Neutrality

$E_F > E_{Fi}$ & $E_F > E_A^{Cr}$.



Need to find N_A^{Cr}, so that $n = 10^9 \text{cm}^{-3}$

@ $n = 10^9 \text{cm}^{-3} = N_C \exp\left(-\frac{E_C - E_F}{kT}\right)$

$$E_F - E_C = -kT \ln \frac{N_C}{n} = -25\,\text{meV} \ln\left(\frac{4.3\times10^{17}\text{cm}^{-3}}{10^9\ \text{cm}^{-3}}\right) \quad ④$$

$$= -\underline{0.497\,\text{eV}} \quad (0.514\,\text{eV if } 25.9\,\text{meV was used})$$

$$E_F - E_A^{Cr} = E_g - (E_C - E_F) - (E_A^{Cr} - E_V)$$

$$= (1.42 - 0.497 - 0.7)\,\text{eV} = 0.223\,\text{eV} \quad (\gg 3kT) \quad ④$$

Cr is fully ionized because E_A^{Cr} is below E_F.

Material neutrality gives: $n = N_D^+ - N_A^- - N_A^{Cr-}$

Main issue:
- used pre-doping level to calculate E_F
- assume ionization without proving.
- flipped the e^- & hole relationship.

$$N_A^{Cr-} = (10^{16} - 2\times10^{15} - 10^9)\,\text{cm}^{-3}$$

$$\simeq \underline{\underline{8.00\times10^{15}\,\text{cm}^{-3}}} \quad (3\text{ s.f.}) \quad ②$$

- n is the majority carrier, not p.

(b) $n = 10^9 \text{cm}^{-3}$

$$p = \frac{n_i^2}{n} = \frac{(2.4\times10^6)^2}{10^9}\text{cm}^{-3} = 5760\,\text{cm}^{-3}. \quad ③ \leftarrow \text{1 for correct } \mu.$$

$$\rho = \frac{1}{q\mu_p p + q\mu_n n} = \frac{1}{(1.6\times10^{-19}\text{C})(6\times10^3\frac{\text{cm}^2}{\text{Vs}})(10^9\text{cm}^{-3})}$$

$q\mu_p p$ negligible

$$= \underline{\underline{1.04\,\text{M}\Omega\cdot\text{cm}}} \quad ①$$

Total doping $= (10^{16} + 2\times10^{15} + 8\times10^{15})\,\text{cm}^{-3} = 2\times10^{16}\,\text{cm}^{-3}$

$$R_\square = \frac{1}{\int_0^{200\mu m} q\mu_n n\,dx} = \frac{\rho}{200\times10^{-4}\text{cm}} \quad ③$$

$$= \underline{\underline{52.1\,\text{M}\Omega}} \quad ①$$

Main issue:
- used pre-doping p to calculate ρ.
- forgot that μs are related to <u>total</u> doping