The test is open book/open notes. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses (50 total).

1. The electron concentration in a region of silicon depends linearly on depth with concentration of $5 \times 10^{15}$ cm$^{-3}$ at surface ($x = 0$) and $10^{15}$ cm$^{-3}$ at depth of $x = 500$ nm. If the vertical electron current density in this region is constant at $J_n = 100$ A/cm$^2$, calculate the electric field near $x = 500$ nm. You may assume that the mobility is constant at 1250 cm$^2$/Vs.

$$J_n = q \left( \mu_n n \Sigma + D_n \frac{dn}{dx} \right)$$

$$\mu_n = 1250 \frac{cm^2}{Vs} \quad D_n = \frac{kT}{q} \mu_n$$

$$J_n = q \mu_n \left( n \Sigma + \frac{kT}{q} \frac{dn}{dx} \right)$$

$$\Sigma = \frac{1}{n} \left[ \frac{J_n}{q \mu_n} - \frac{kT}{q} \frac{dn}{dx} \right] = \frac{1}{10^{15} \text{cm}^{-3}} \left[ \frac{100 \text{A/cm}^2}{(1.6 \times 10^{-19} \text{C}) (1250 \frac{\text{cm}^2}{\text{Vs}})} \right]$$

$$= 10^{-15} \text{cm}^3 \left[ \frac{100 \text{ V/cm}^4}{2 \times 10^{-16}} - 2.08 \times 10^{18} \frac{\text{V}}{\text{cm}^4} \right]$$

$$= 2.58 \times 10^3 \frac{\text{V}}{\text{cm}} = 2580 \frac{\text{V}}{\text{cm}}$$
2. (a) Sketch the band diagram ($E$ vs. $k$ along main symmetry directions) for a semiconductor which has $\mu_p > \mu_n$ and $N_v < N_c$. The qualitative relations should be clear from your sketch. Assume scattering lifetimes are equal for holes and electrons. Briefly note reasoning. (10)

\[ \mu_p = \frac{g_p}{m^*_h} \quad \mu_n = \frac{g_n}{m^*_e} \quad \mu_p > \mu_n \Rightarrow \frac{1}{m^*_h} > \frac{1}{m^*_e} \]

\[ \frac{\tau_p}{\tau_n} \Rightarrow \left( \frac{dE}{dk^2} \right)_{\text{VB}} > \left( \frac{d^2E}{dk^2} \right)_{\text{CB}} \]

\[ N_v = 2N_{\text{max}} \left( \frac{2\pi m^*_h kT}{\hbar^2} \right)^{3/2} \]

\[ N_v < N_c \Rightarrow m^*_h < m^*_e \quad \text{and/or} \quad N_{\text{max}}^{\text{VB}} < N_{\text{min}}^{\text{CB}}. \]

(b) Calculate the intrinsic carrier concentration at 300 K in a semiconductor which has a single valence band maxima given by $E = E_v - (\hbar^2 k^2 / 2m_0)$ and eight conduction band minima in the $\langle 111 \rangle$ directions with $E = E_c + 2\hbar^2 (k - k_0)^2 / m_0$ where $k_0 = (\pm a, \pm a, \pm a)$, $E_c - E_v = 1$ eV and $m_0$ is the free electron mass. (10)

\[ n_i = \sqrt{N_c N_v} \exp \left( - \frac{E_g}{2kT} \right) \]

\[ E_g = E_c - E_v = 1 \text{ eV} \]

\[ N_c = 2N_{\text{min}} \left( \frac{2\pi m^*_e kT}{\hbar^2} \right)^{3/2} \]

\[ m^*_e = \frac{\hbar^2}{4\hbar^2/m_0} = \frac{\hbar^2}{4\hbar^2/m_0} = \frac{m_0}{4} \]

\[ N_v = 2 \left( \frac{2\pi m^*_h kT}{\hbar^2} \right)^{3/2} \]

\[ m^*_h = m_0 \]

\[ N_i = 2 \left( \frac{2\pi m_0 kT}{\hbar^2} \right)^{3/2} \left[ \left( \frac{1}{4} \right)^{3/2} \left( \frac{1}{3} \right) \right]^{1/2} \exp \left( - \frac{1 \text{ eV}}{0.052 \text{ eV}} \right) \]

\[ = 2 \left( \frac{2\pi \left( 9.11 \times 10^{-31} \text{ kg} \right) (0.026 \text{ eV})}{(6.63 \times 10^{-34} \text{ kg} \cdot \text{m} \cdot \text{s}) (4.14 \times 10^{-15} \text{ eV} / \text{s})} \right)^{3/2} \times 4.45 \times 10^{-9} = 1.12 \times 10^{17} \text{ m}^{-3} \]

\[ = 1.12 \times 10^{11} \text{ cm}^{-3} \]
3(a) \( N_D = 10^{16} \text{ cm}^{-3} \) \( N_A = 2 \times 10^{15} \text{ cm}^{-3} \) \( \rightarrow \) assume all shallow dopants \( \rightarrow \) fully ionized. 

Steps:

- Proof of assumptions

Need to find \( N_A^{cr} \), so that \( n = 10^9 \text{ cm}^{-3} \)

Neutral \( @ \ n = 10^9 \text{ cm}^{-3} \)

\[
E_F - E_c = -kT \ln \frac{N_c}{n} = -25 \text{ meV} \ln \left( \frac{4.5 \times 10^{17} \text{ cm}^{-3}}{10^9 \text{ cm}^{-3}} \right) 
= -0.497 \text{ eV} \quad (0.514 \text{ eV} \\
\text{if} \ 25.9 \text{ meV was used})
\]

\[
E_F - E_A^{cr} = \frac{E_g}{2} - (E_c - E_F) - (E_A^{cr} - E_V) \\
= (1.42 - 0.497 - 0.7) \text{ eV} = 0.223 \text{ eV} \quad (\gg 3kT)
\]

\( E_A^{cr} \) is fully ionized because \( E_A^{cr} \) is below \( E_F \).

Material neutrality gives:

\( n = N_D^+ - N_A^- - N_A^{cr} \)

Main issue: used pre-doping level \( N_A^{cr} = (10^{16} - 2 \times 10^{15} - 10^9) \text{ cm}^{-3} \)

to calculate \( E_F \)

- Assume ionization without proving.

\( n = 8.00 \times 10^{15} \text{ cm}^{-3} \) (3 s.f.)

(b) \( n = 10^9 \text{ cm}^{-3} \) \( \rightarrow \) \( n \) is the majority carrier, not \( p \).

\[
P = \frac{n^2}{n} = \frac{(2.4 \times 10^6)^2}{10^9} = 5760 \text{ cm}^{-3}.
\]

\[
P = \frac{1}{\frac{q}{n^2} \mu_p + \frac{1}{q} \mu_n} = \frac{4.6 \times 10^{-14} \text{ cm}^2}{(6 \times 10^3 \text{ cm}^2) (10^9 \text{ cm}^{-3})} \\
\text{negligible} = 1.04 \text{ M} \Omega \cdot \text{cm}
\]

\[
R_D = \frac{1}{\frac{2}{q} \mu_n \mu_{max}} = \frac{1}{280 \times 10^{-7} \text{ cm}} = 52.1 \text{ M} \Omega
\]

Main issue: used pre-doping \( p \) to calculate \( p \).

- Forgot that \( n \) and \( p \) are related to total doping.