Homework #1 - EE 482

due 1/10/11

1. (a) The unit "electron-volt" (eV) represents the energy gained by an electron (charge $-q$) falling through a 1V potential difference in a vacuum. What is 2 eV in Joules?

   (b) Calculate the wavelength of

   i. a 0.04 eV free electron

   ii. a 1.2 eV photon

   iii. a fastball (45 m/s, 145 grams)

   $$m_{\text{electron}} = 9.1 \times 10^{-31}\,\text{kg}, \qquad q = 1.6 \times 10^{-19}\,\text{C}$$

2. Consider an electron in an infinite potential well. Assume that the well has energy $V = 0$ for $0 < x < a$.

   (a) Calculate and sketch the wavefunctions for the lowest three energy levels. Determine the expected value of x, p_x (momentum) and E (energy) for each of these states.

   (b) The standard deviation of the position is $\langle \Delta x \rangle = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$. A similar expression holds for $\langle \Delta p_x \rangle$. If the operator for ${p_x}^2$ is $-\hbar^2 \partial^2/\partial x^2$, check the Heisenberg uncertainty principle ($\Delta x \Delta p_x \geq \hbar/2$) for the lowest energy state.

3. A band in an imaginary two dimensional crystal is given by

   $$E = A({k_x}^2 + 2{k_y}^2) - B(3{k_x}^4 + {k_y}^4),$$

   with A and B positive constants.

   (a) Find the wavevectors and corresponding energies at which the electron velocity is zero. Which of these states represent maxima or minima of the energy band. Draw a rough sketch of the E versus k diagram.

   (b) Find the effective mass tensor for electrons in each of the states found in (a). Which of these states will result in purely "electron-like" behavior and which in purely "hole-like" behavior? How can you see this from the sketch of part(a).

   (c) If a force in the (11) direction is applied to an electron with a wavevector $(k_x, k_y) = (0, 0)$, in what direction will the electron be accelerated?

   (d) Repeat (c) for one of the band maxima.

4. The outer occupied energy band in a crystal with N atoms per unit volume is described by

   $$E = E_0 + \frac{\hbar^2 (k - k_0)^2}{2m^*}$$

   and the band contains one electron per atom.

   (a) Is this material an insulator, a metal or a semiconductor? Explain.

   (b) Give an expression for $E_f - E_0$ at 0K in terms of m^*?