Homework #2 Solutions
EE 528, Spring 2015

Problem 1 (PDG 3.3). A Czochralski crystal is pulled from a melt containing 10^{15} cm^{-3} boron and $2x10^{14}$ cm^{-3} phosphorus. Initially the crystal will be P type but as it is pulled, more and more phosphorus will build up in the liquid because of segregation. At some point the crystal will become N type. Assuming $k_O = 0.32$ for phosphorus and 0.8 for boron, calculate the distance along the pulled crystal at which the transition from P to N type takes place.

Answer:

We can calculate the point at which the crystal becomes N type from Eqn. 3.38 as follows:

$$C_S\,(\text{Phos}) = C_0 k_0 (1-f)^{k_0 - 1} = \left(2x10^{14}\right)(0.32)(1-f)^{-0.68}$$

$$C_S\,(\text{Boron}) = C_0 k_0 (1-f)^{k_0 - 1} = \left(10^{15}\right)(0.8)(1-f)^{-0.2}$$

At the point where the cross-over occurs to N type, these two concentrations will be equal. Solving for f, we find

$$f \cong 0.995$$

Thus only the last 0.5% of the crystal is N type.

2(a) $S_{mix}^{total} = k \ln w$

$$= k \ln\left(\frac{N!}{(N-n)!\,n!}\right) = k \ln\left(\frac{1{,}000{,}000!}{980{,}000!\,20{,}000!}\right)$$

Using Stirling's approximation (good for large x)

$$\ln(x!) \simeq x \ln x - x$$

$$S_{mix}^{total} \simeq k\left[\ln(10^6!) - \ln(980000!) - \ln(20{,}000!)\right]$$

$$= k\left[10^6 \ln 10^6 - 980000 \ln(980000) - 20000 \ln(20000)\right]$$

$$= 9.8 \times 10^4\, k = 8.45\ eV/K$$

$$\Delta S_{mix} = S_{mix}^{total}(2\times10^4) - S_{mix}^{total}(9999)$$

$$= k \ln\left(\frac{10^6!}{980000!\,20000!}\right) - k \ln\left(\frac{10^6!}{980001!\,19999!}\right)$$

$$= k \ln\left(\frac{980001}{20000}\right) = 3.9\, k = 3.35 \times 10^{-4}\ \frac{eV}{K}$$

Note that $\frac{980001}{20000} = \frac{N-n+1}{n} \simeq \frac{C_S - C_A}{C_A}$

as derived for N large in class

b) $C_I^* = \theta_I C_S \exp\left(-\frac{\Delta G_I^f}{kT}\right) = \begin{cases} 10^{12}\ cm^{-3} @ 1000°c \\ 2.6\times10^{9} cm^{-3} @ 800°c \end{cases}$

$\theta_I = 6 \quad \Delta G_I^f = kT \ln\left(\frac{\theta_I C_S}{C_I^*}\right) =$

$= \left(8.62\times10^{-5}\frac{eV}{K}\right)\left(273 + T(°c)\right)\ln\left(\frac{\theta_I C_S}{C_I^*}\right)$

$= \begin{cases} 2.90\ eV \quad @ 1000°C\ (1273K) \\ 2.995 eV \quad @ 800°C\ (1073K) \end{cases}$

$\Delta G_I^f = \Delta H_I^f - T\Delta S_I^f \Rightarrow \Delta S_I^f = 4.75\times10^{-4}\frac{eV}{K} = 5.5k$

$\Delta H_I^f = 3.505\ eV$

3. (a) $\dfrac{C_{V^-}}{C_{V^0}} = \dfrac{\theta_{V^-}}{\theta_{V^0}} \exp\left(-\dfrac{E_{V^-}-E_{fi}}{kT}\right)$ Assume all $\theta_{v,i}$'s equal

In intrinsic material:

$$= \exp\left(-\frac{(E_v+0.77-0.57)-(E_v+0.33)}{kT}\right)$$

$E_g(1273K) = 0.77eV$ from Eq 1.18

$$= \exp\left(-\frac{0.20-0.33}{0.11}\right) = 3.26$$

$$\frac{C_{V^=}}{C_{V^0}} = \frac{C_{V^=}}{C_{V^-}}\frac{C_{V^-}}{C_{V^0}} = \exp\left(-\frac{0.66-0.33}{0.11}\right)\cdot 3.26 = 0.16$$

$$\frac{C_{V^+}}{C_{V^0}} = \exp\left(\frac{0.05-0.33}{0.11}\right) = 0.078$$

$$\frac{C_{V^{++}}}{C_{V^0}} = \frac{C_{V^{++}}}{C_{V^+}}\frac{C_{V^+}}{C_{V^0}} = \exp\left(\frac{0.13-0.33}{0.11}\right)0.078 = 0.013$$

$$C_{V\,total}^{i} = C_{V^0}^{i} + C_{V^-}^{i} + C_{V^=}^{i} + C_{V^+}^{i} + C_{V^{++}}^{i}$$

$$10^{11}\,cm^{-3} = C_{V^0}^{i}\left[1+3.26+0.16+0.078+0.013\right] = 4.51\,C_{V^0}$$

$$C_{V^0} = \frac{10^{11}\,cm^{-3}}{4.51} = \underline{\underline{2.2\times10^{10}\,cm^{-3}}}$$

(b) $C_{V\,total} = C_{V^0} + C_{V^-}^{i}\left(\frac{n}{n_i}\right) + C_{V^=}^{i}\left(\frac{n}{n_i}\right)^2 + C_{V^+}^{i}\left(\frac{n_i}{n}\right) + C_{V^{++}}^{i}\left(\frac{n_i}{n}\right)^2$

$n_i \cong 9\times10^{18}\,cm^{-3}$ @ $1000°C$ from plot in Plummer, Deal & Griffin page 16.

$$\left(\frac{n}{n_i}\right) = \frac{5\times10^{19}}{1.8\times10^{19}} + \sqrt{\left(\frac{5}{1.8}\right)^2+1} = 5.73 \quad \left[\frac{N_i^+}{n_i} = 5.55\right]$$

$$C_V^{total}\left(N_d^+ = 5\times10^{19}\,cm^{-3}\right) = 2.2\times10^{10}\,cm^{-3}\left[1+(3.26)(5.73)+(0.16)(5.73)^2 + (0.078)/5.73 + \frac{0.013}{(5.73)^2}\right]$$

$$= \underline{\underline{5.5\times10^{12}\,cm^{-3}}}$$

~~For acceptors (N_a^- instead of donors)~~

$$\left(\frac{n}{n_i}\right) = \frac{1}{5.73} = 0.175$$

$$C_V^{total}\left(N_a^- = 5\times10^{19}\,cm^{-3}\right) = 2.2\times10^{10}\,cm^{-3}\left[1+\frac{3.26}{5.73}+\frac{0.16}{(5.73)^2}+0.078(5.73)+0.013(5.73)^2\right]$$

$$= 5.4\times10^{10}\,cm^{-3}$$

(c)