The test is open book/open notes. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 100 points in 4 problems on 4 pages.

1. A semiconductor has eight conduction band minima given by \( E = E_c + A(k - k_i)^2 \), where \( A \) is a positive constant and \( k_i = (\pm k_0, \pm k_0, \pm k_0) \).

   (a) Give expression for the conduction band effective density of states \( (N_c) \). (10)

   \[
   N_c' = 2 \left( \frac{2\pi m_e^* kT}{\hbar^2} \right)^{3/2} = 2 \left( \frac{\pi}{4\pi^2 A} \right)^{3/2} \frac{kT}{\hbar^2} = 2 \left( \frac{kT}{4\pi^2 A} \right)^{3/2} = \frac{1}{4} \left( \frac{kT}{\pi A} \right)^{3/2}
   \]

   8 minima
   \[
   N_c = 8N_c' = 2 \left( \frac{kT}{\pi A} \right)^{3/2}
   \]

   (b) If the scattering rate is \( S(k,k') = S_0 \delta[E(k) - E(k')] \), what would be the momentum relaxation time as function of \( E \)? (10)

   Isotropic elastic scattering, so expected value of \( k = 0 \) after scattering

   \[
   \frac{1}{\tau_m} = \frac{1}{\tau} = \int S(k,k') dk' \quad \text{8 minima}
   \]

   \[
   S_0 g_c(E) = S_0 \left[ 8 \left( \frac{4\pi}{\hbar^3} \right)^{3/2} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} \left( E - E_c \right)^{1/2} \right] \hspace{1cm} \text{8 minima}
   \]

   \[
   = S_0 32\pi \left( \frac{2}{8\pi^2 A} \right)^{3/2} \left( E - E_c \right)^{1/2}
   \]

   \[
   \frac{1}{\tau_m} = S_0 \frac{4}{\pi} A^{-3/2} \left( E - E_c \right)^{1/2} \hspace{1cm} \tau_m(E) = \frac{4}{\pi S_0} A^{3/2} \left( E - E_c \right)^{-1/2}
   \]
2. A silicon sample is uniformly doped with \( N_a = 10^{16} \text{cm}^{-3} \). The sample is 0.2\( \mu \text{m} \) thick. At one surface (\( x = 0 \)), carriers are generated at a rate of \( 10^{19} \text{cm}^{-2} \text{s}^{-1} \) (note: this is surface generation and is per unit area). There is no bulk generation. At both surfaces (\( x = 0 \) and \( x = 0.2 \mu \text{m} \)), there is a recombination velocity of \( s = 10^6 \text{cm/s} \). Assume \( \tau_n = \tau_p = 1 \mu \text{s} \), \( D_n = 25 \text{cm}^2/\text{s} \), and \( D_p = 9 \text{cm}^2/\text{s} \).

(a) Determine the excess electron concentration as a function of position. (14)

\[ L_n = \sqrt{D_n \tau_n} = 50 \mu \text{m} \gg 0.2 \mu \text{m} \implies \text{like short base, very little recombination in bulk} \]

\[ \Delta n = A x + B \]

Assume LLI, use Diff. Approx.

At \( x = 0 \); \( G_{ls} = s \Delta n(0) - D_n \frac{d \Delta n}{dx} = s B - D_n A \)

At \( x = 0.2 \mu \text{m} \); \( 0 = s \Delta n(0.2 \mu \text{m}) + D_n \frac{d \Delta n}{dx} = s (A \times 2 \times 10^{-5} \text{cm} + B) + D_n A \)

\[ 10^{19} = 10^6 B - 25A \implies B = 10^{13} + 2.5 \times 10^{-5}A \]

\[ 0 = 10^6 (A \times 2 \times 10^{-5} + 10^{13} + 2.5 \times 10^{-5}A) + 25A \]

\[ 0 = 45A + 10^9 + 25A \implies A = -\frac{10^{19}}{70} = -1.43 \times 10^{17} \text{cm}^{-4} \]

\[ B = 10^{13} - 3.57 \times 10^{12} \]

\[ = 6.43 \times 10^{12} \text{cm}^{-3} \]

\[ \Delta n = 6.43 \times 10^{12} \text{cm}^{-3} - 1.43 \times 10^{17} \text{cm}^{-4} x \]

\[ \Delta n \ll N_a, \text{ so LLI, Diff Approx OK} \]

(b) Calculate the two components of hole current density near \( x = 0 \). (14)

\[ \Delta p = \Delta n \ (\text{Quasi-neutral}) \]

\[ J_p = -q D_p \frac{d \Delta p}{dx} = -q D_p A = 0.22 \text{A/cm}^2 \]

\[ J = J_n + J_p = 0 \]

\[ J_{\text{total}} = -J_{\text{diff}} = -J_n = -q D_n A \]

\[ J_{\text{drift}} = J_{\text{total}} - J_{\text{diff}} = -q D_n A + q D_p A = -q (D_n - D_p) A \]

\[ J_{\text{drift}} = 0.38 \text{A/cm}^2 \]
3. An abrupt Ge/GaAs heterojunction is fabricated with the Ge \((E_g = 0.67 \text{V}, \chi = 4.0 \text{V}, N_e = 10^{19} \text{cm}^{-3}, N_v = 5.4 \times 10^{18} \text{cm}^{-3}, K_s = 16.0)\) doped p-type with \(N_a = 10^{17}\) and the GaAs \((E_g = 1.42 \text{V}, \chi = 4.07 \text{V}, N_e = 4.2 \times 10^{17} \text{cm}^{-3}, N_v = 9.5 \times 10^{18} \text{cm}^{-3}, K_s = 13.1)\) doped n-type with \(N_d = 10^{17}\). Assume that the number of free carriers near the interface can be neglected, but that there is a fixed density of interface charges equal to \(Q_{\text{int}}/q = 5 \times 10^{11} \text{cm}^{-2}\).

(a) Calculate the built-in voltage. (7) \[
\Phi_{bi} = \Phi_Ge - \Phi_{GaAs} = 0.46 \text{V}
\]
\[
\Phi_Ge = \chi_{Ge} + E_g^{Ge} - \frac{kT}{q} \ln \left( \frac{N_v}{N_a} \right) = 4.0 + 0.67 - 0.10 = 4.57 \text{V}
\]
\[
\Phi_{GaAs} = \chi_{GaAs} + \frac{kT}{q} \ln \left( \frac{N_e}{N_d} \right) = 4.07 + 0.04 = 4.11 \text{V}
\]

(b) For \(V_A = 0\), calculate the peak electric field and sketch the electric field versus position. (18)

\[
\varepsilon_{\text{max}} = \frac{q \times p N_a}{\varepsilon_0} = -1.13 \times 10^{16} \text{V}\text{cm}^{-2}
\]
\[
applied V = K_{Ge} \varepsilon_0 = 16 \rightarrow K_{Ge} \varepsilon_0
\]
\[
\varepsilon_1 = - \frac{q \times p N_a - Q_{\text{int}}}{K_{GaAs} \varepsilon_0} = -1.38 \times 10^{16} \text{V}\text{cm}^{-2} + 6.9 \times 10^4
\]
\[
X_n = \frac{x_p N_a - Q_{\text{int}}}{N_d} = x_p - 5 \times 10^{-6} \text{cm}
\]
\[
\Phi_{bi} - V_A = -\left[ \frac{1}{2} \varepsilon_{\text{max}} x_p + \frac{1}{2} \varepsilon_1 x_n \right]
\]
\[
0.46 \text{V} = \frac{1.13 \times 10^{16} x_p^2 V}{2} + \left( 6.9 \times 10^4 \right) (x_p - 5 \times 10^{-6})
\]
\[
\rightarrow 0.46 = 56.5 x_p^2 + (69 x_p - 6.9) (x_p - 0.05)
\]
\[
125.5 x_p^2 - 10.35 x_p - 0.115 = 0
\]
\[
X_p = \frac{10.35 \pm \sqrt{(10.35)^2 + 4 \times 125.5 \times 0.115}}{2 \times 125.5} = 0.092 \text{um} = 92 \text{nm}
\]
\[
\Rightarrow \varepsilon_{\text{max}} = -1.04 \times 10^5 \text{V}\text{cm}^{-1}, \varepsilon_1 = -5.8 \times 10^4 \text{V}\text{cm}^{-1}
\]
4. (a) Calculate the average electron kinetic energy in a nondegenerate 2D electron gas. [Hint: Consider first a single sub-band and note energy in excess of sub-band minimum is kinetic energy.] (10)

\[ g_{2D} = \frac{m^*}{\hbar^2} \text{ constant} \]

\[ E > E_n \]

\[ \langle KE \rangle = \int_{E_n}^{\infty} (E - E_n) g_2(E) f(E) \, dE \]

\[ \frac{\int_{E_n}^{\infty} g_2(E) f(E) \, dE}{\int_{E_n}^{\infty} f(E) \, dE} \]

\[ h^n \rightarrow \int_{E_n}^{\infty} g_2(E) f(E) \, dE \]

\[ \# \text{ of electrons in } n\text{-th sub-band} \]

\[ \tilde{\gamma} = E - E_n \]

(b) For a metal-silicon contact with a large density of surface states, why might you expect adding Ge near surface to reduce contact resistance? (7)

Assume Fermi-level pinned near midgap

\[ \phi_n = -\frac{1}{q} (E_c - E_i) = \frac{E_g}{2q} \quad \implies \phi \psi \implies J \mathbf{i} \]

\[ \phi_p = -\frac{1}{q} (E_v - E_i) = \frac{E_g}{2q} \quad \implies R \mathbf{i} \]

(c) If \( \langle \tau_E \rangle = (E/\hbar \omega_0)(\tau_m) \), determine an expression for the steady-state drift current in homogenous material based on the hydrodynamic equations. [Note this approximates a system dominated by optical phonon scattering.] (10)

\[ \frac{d\langle p \rangle}{dt} = 0 = -q \langle E \rangle - \frac{\langle p \rangle}{\tau_{m}} \quad \text{in homogeneous system} \quad \text{(all spatial } \nabla E \text{ terms)} \]

\[ \frac{d\langle E \rangle}{dt} = 0 = -q \langle E \rangle - \frac{q^2}{m^*} \langle p \rangle \langle E \rangle \]

\[ \langle E \rangle = -q \frac{3}{3} \langle p \rangle \langle E \rangle = \frac{q^2}{m^*} \langle p \rangle \langle E \rangle \]

\[ \langle E \rangle = -q \frac{3}{3} \langle p \rangle \langle E \rangle = \frac{q^2}{m^*} \langle p \rangle \langle E \rangle \]

\[ \langle p \rangle = -\left( \frac{m^* \hbar \omega_0}{12} \right) \]

\[ \langle E \rangle = -q \frac{3}{3} \langle p \rangle \langle E \rangle = \frac{q^2}{m^*} \langle p \rangle \langle E \rangle \]

End Of Exam