Exam #2 — EE 531  
Winter 2003

The test is open book/open notes. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 140 points in 6 problems on 6 pages. Assume $T = 300K$.

1. An MOS capacitor is made with a $p^+$ poly gate ($E_f = E_v$), a uniformly doped substrate with $N_d = 2 \times 10^{18} \text{cm}^{-3}$ and $t_{ox} = 2 \text{nm}$. There is a substantial density of surface states such that $N_{ss}(E) = 10^{12} \text{cm}^{-2} \text{eV}^{-1}$ and $Q_{ss} = 0$ when $E_f = E_i$.

(a) If $\psi_s = -0.2 \text{V}$, calculate the applied gate voltage. (14)

$$\frac{dQ_{ss}}{dE_f} = -qN_{ss}(E)$$

In bulk ($E_f - E_i$) = $kT \ln \left( \frac{N_d}{n_i} \right) = 0.495 \text{eV}$

$-\phi_F < \psi_s < 0 \Rightarrow$ depletion. At surface, $E_f - E_i = 0.495 - 0.2 = 0.295 \text{eV}$

$$Q_{ss} = \int_{E_i}^{E_f} N_{ss}(E) dE = -q \left( 10^{12} \text{cm}^{-2} \text{eV}^{-1} \right) (0.295 \text{eV}) = -4.72 \times 10^{-8} \text{C/cm}^2$$

$$Q_s = Q_d = \sqrt{2K_s\varepsilon_0 qN_d / \psi_s} = 3.66 \times 10^{-7} \text{C/cm}^2$$

$$C_{df} = \frac{K_s \varepsilon_0 E_i}{x_{df}} = 1.7 \times 10^{-6} \text{F/cm}^2$$

$$\Phi_{ms} = (\psi_s + E_f) - (\psi_s + \frac{kT}{q} \ln \left( \frac{N_c}{N_d} \right)) = 1.05 \text{V}$$

$$\psi_g = \Phi_{ms} + \psi_s + \psi_{df} = 1.05 \text{V} + (-0.2 \text{V}) - \frac{(3.66 \times 10^{-7} \text{C/cm}^2 - 4.72 \times 10^{-8} \text{C/cm}^2)}{1.7 \times 10^{-6} \text{F/cm}^2}$$

$$\psi_g = 0.46 \text{V}$$

(b) What is the low frequency small signal capacitance that would be measured under these conditions (don’t forget the interface charges)? (10)

$$C_{it} = \left| \frac{dQ_{ss}}{d\psi_s} \right| = qN_{ss}(E) = 1.6 \times 10^{-19} \text{C} \times 10^{12} \text{cm}^{-2} \text{eV}^{-1} = 1.6 \times 10^{-7} \text{F/cm}^2$$

$$C_d = \sqrt{\frac{K_s \varepsilon_0 qN_d}{2\psi_s}} = 9.14 \times 10^{-7} \text{F/cm}^2$$

$$C = \frac{1}{C_{df}} + \frac{1}{C_{it} + C_d} = 6.6 \times 10^{-7} \text{F/cm}^2$$
2. An nMOS transistor has $V_{FB} = -1.05V$ and $t_{ox} = 2\text{nm}$. The channel doping is small up to a depth $a$ and then the doping rises abruptly to $4 \times 10^{18} \text{cm}^{-3}$. Assume $V_{SB} = 0$.

(a) What value of $a$ would give a long-channel threshold voltage of 0.5 V? (15)

\[
V_{th} = V_{FB} - 2\Phi_F - \frac{Qd_{\text{max}}}{C_{ox}}
\]

\[
0.5 = -1.05 + 1.02 - \frac{Qd_{\text{max}}}{1.7 \times 10^{-6} \text{F/cm}^2}
\]

\[
\Rightarrow Qd_{\text{max}} = 9.0 \times 10^{-7} \text{C/cm}^2
\]

\[
\Phi_F = -\frac{2V}{9} \ln \left( \frac{4 \times 10^{10}}{10^{10}} \right) = -0.51 \text{V}
\]

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\]

\[
\Rightarrow \frac{9 \times 10^{-7}}{\text{C/cm}^2} \cdot \frac{2 \times 10^{-7}}{\text{C/cm}^2} = k(x_d - a)
\]

\[
x_d = 1.9 \times 10^{-6} \text{cm}
\]

\[
a = 0.5 \times 10^{-6} \text{cm}
\]

(b) If $V_{gs} = 1V$, calculate $V_{ds}^{sat}$. Plot the charge density, electric field and band diagram versus vertical position in the channel close to the drain under these conditions. (16)

\[
V_{gs} = 1V
\]

\[
V_{ds}^{sat} = \frac{V_{gs} - V_{th}}{m}
\]

\[
m = 1 + \frac{3 \times 10^{-7}}{1.9 \times 10^{-6}} = 1 + \frac{3 \times 10^{-7}}{1.9 \times 10^{-6}} = 1.32
\]

\[
\frac{1 - 0.5}{1.32} = 0.38V
\]

EOSI, negligible $Q_d$.
3. An nMOS transistor with uniform channel doping has $V_{FB} = -1.05V \pm 0.03$, $t_{ox} = 2 \pm 0.1\text{nm}$, $N_a = 2 \pm 0.1 \times 10^{18}\text{cm}^{-3}$, $W = 0.15 \pm 0.01\mu\text{m}$, and $L_{eff} = 0.07 \pm 0.01\mu\text{m}$ and is operated in system with $V_{dd} = 1.2 \pm 0.1V$. Assume mobility in inversion layer is given by value at edge of strong inversion.

What is the worst-case leakage current ($V_{gs} = 0$, $0 \leq V_{ds} \leq V_{dd}$)? Note that process variations can have multiple effects. Identify dominant effect in determining worst-case condition. (25)

Worst case is: $V_{FB} = -1.08V$ (lowest $V_{th}$) $C_{ox} = 1.8 \times 10^{-6} \frac{F}{\text{cm}^2}$

$6_{ox} = 1.9 \text{nm}$ (largest $C_{ox}$, smallest $V_{th}$)

Thin oxide gives better subthreshold slope but for given surface potential, $Q_s > 0$, so largest $C_{ox}$ gives lowest $V_{g}$

$\phi_F = \frac{kT}{q} \ln \left( \frac{1.9 \times 10^{18}}{10^6} \right) = -0.444V$

$G_{d_{max}} = \frac{2 \varepsilon L_{eff} N_a (2 \varepsilon)}{q} = 7.9 \times 10^{-7} \frac{\text{C}}{\text{cm}^2}$

$N_a = 1.9 \times 10^{18} \text{cm}^{-3}$ (smallest $V_{th}$)

$W = 0.16 \mu\text{m}$ (largest current)

$L_{eff} = 0.06 \mu\text{m}$ (Smallest $V_{th}$ due to SCE, also larger $W/L$)

$V_{ds} = V_{dd} = 1.3V$

$V_{th}^0 = -1.08 + 0.988 + \frac{7.9 \times 10^{-7}}{1.8 \times 10^{-6}} = 0.34V$ $\psi_{bi} = \frac{E_F}{2q} + 1.9kT$

$\Delta V_t = 8(m-1) \sqrt{\psi_{bi} (\psi_{bi} + V_{ds})} e^{-\pi L/2mW_{dm}} = 1.05V$

$W_{dm} = \frac{12L_{eff}}{q N_d} = 2.6 \times 10^6 \text{cm}$

$m-1 = \frac{3 + \psi_{th}}{W_{dm}} = 0.22$

$V_{th} = 0.2V$ $\Sigma_{eff} = \frac{V_t + 0.2}{3 + \psi_{th}} + \frac{V_g - V_t}{6 + \psi_{th}}$

$= 8.8 \times 10^5 \frac{V}{\text{cm}} \implies M_{eff} \approx 200$

$I_{ds} = M_{eff} \frac{W}{L} (m-1) \left( \frac{kT}{q} \right)^2 e^{\frac{\psi_{bi}}{kT}} \left( \frac{V_g - V_t}{kT} \right) \left( 1 - e^{\frac{V_{gs}}{kT}} \right)$

$= (200) \left( 1.8 \times 10^{-6} \frac{F}{\text{cm}^2} \right) \left( 0.16 \right) \left( 0.22 \right) \left( 0.0259V \right)^2 \exp \left( \frac{-0.2}{0.0259} \right) = 6.3 \times 10^{-11} A$
4. We will compare the behavior of two similar silicon nMOS (p-substrate) devices at 300K. One of the devices is a standard structure with channel region uniformly doped. The other has slightly lower bulk doping and an addition shallow acceptor implant. For both transistors, the oxide thickness, channel dimensions \((W, L)\), and threshold voltage are the same.

(a) Which has the preferable subthreshold slope factor? Explain. (8)

(b) Which device has the lower drain to body junction capacitance? Explain. (7)

(c) Which transistor will be more immune to reductions in threshold voltage due to short channel effects? Explain. (7)
5. Consider a long-channel MOSFET transistor an \( n^+ \) polysilicon gate. The channel region is doped uniformly depth, but the doping varies laterally from the source to the drain as:

\[
N_A = N_A^S, \text{ for } 0 < y < L/2
\]

\[
N_A = N_A^D < N_A^S, \text{ for } L/2 < y < L
\]

(a) Give an expression for the threshold voltage. (8)

\[
V_{tH} = V_{th} (N_A = N_A^S) = \phi_{MS} - 2 \phi_F - \frac{Q_{d, \text{max}}}{q} \left( \frac{1}{\cos \theta} \right)
\]

\[
\phi_F = - \frac{kT}{q} \ln \left( \frac{N_A^S}{n_i} \right), \quad Q_{d, \text{max}} = \sqrt{2kT \varepsilon_0 q N_A^S 12 \phi_F 1}
\]

(b) Derive an expression for the linear drain current. (16)

Like two devices in series each \( W/L = \frac{L}{2} \) and different \( V_T, \) m

\[
V_S = V_S^T, \quad V_G = V_G^T = V_S^Z = V_D^2 = V_D
\]

\[
V_D = V_S^2 = V_H^2, \quad V_H^2 = V_S^2
\]

\[
I_{DS} = I_{DS}^T
\]

\[
\frac{W}{L} \frac{N_{eff}}{j' \alpha} \left[ (V_G - V_T) V_H^2 - \frac{m_s V_H^2}{2} \right] = \frac{W}{L} \frac{N_{eff}}{j' \alpha} \left[ (V_G - V_H - V_D^2)(V_H - V_D^2) \right]
\]

\[
(V_G - V_T) V_H^2 - \frac{m_s V_H^2}{2} = (V_G - V_H - V_D^2)(V_H - V_D^2) - \frac{m_s (V_H - V_D^2)}{2}
\]

\[
0 = \left( 1 - \frac{m_s^2}{2} \right) (V_H^2) - (V_G - V_T + V_G - V_H - m_s V_D) V_H^2 + (V_G - V_H - m_D^2 V_D) V_D^2
\]

\[
V_H^2 = -b \pm \sqrt{b^2 - 4ac} \quad \frac{2}{2a}
\]

Substitute in \( I_{DS}^T \) or \( I_{DS}^T \) equation

\[
V_T^S = V_T (N_A^S, V_{SB} = 0), \quad V_T^D = V_T (N_A^D, V_{SB} = V_H^2)
\]

\[
m_s = m(N_A^S, V_{SB} = 0), \quad m_D = m(N_A^D, V_{SB} = V_H^2)
\]
6. It can be seen from the plots of mobility versus effective vertical field that the mobility increases and then decreases as the effective field increases.

(a) Why does the mobility drop for high vertical fields and why is the mobility nearly the same for all doping levels under these conditions? (7)

For high vertical fields, carriers are closer to surface and thus surface scattering becomes stronger. Since surface scattering is independent of doping level, all mobilities become equal. Also as $E_F$ increases, the surface carrier concentration increases, so screening reduces ionized impurity scattering.

(b) Why does the mobility also drop as the field is reduced, particularly for heavily-doped channels? Note that the mobility drops substantially below the bulk mobility for a given doping. Hint: Think about the primary scattering mechanism limiting mobility under these conditions and how it changes as $E_F - E_v$ changes near interface. (7)

As $E_F$ drops, the surface carrier concentration is less so the device becomes depleted rather than inverted. The result is less screening and thus stronger ionized impurity scattering, dropping mobility.

End Of Exam