1. In a real semiconductor, trapping centers have a variety of energies within the bandgap. Consider a silicon sample with \( N_1 = 10^{12} \text{cm}^{-3} \) recombination centers due to metal contamination located at the intrinsic Fermi level, and \( N_2 = N_d \) recombination centers due to shallow dopants located 0.05 eV below the conduction band. Assume that the electron and hole thermal velocities are \( 10^7 \text{cm/s} \) and \( 6 \times 10^6 \text{cm/s} \), respectively.

(a) Assuming that recombination via the shallow and deep levels occurs independently, and that capture cross sections for both shallow and deep centers are \( \sigma = 10^{-14} \text{cm}^2 \), calculate the minority carrier lifetime at 300K under low-level injection for \( N_d = 2 \times 10^{18} \text{cm}^{-3} \). Do the shallow or the deep centers dominate the recombination lifetime?

(b) In reality, the capture cross-section for shallow dopants is much smaller than for deep levels. Assuming that the upper line on the curve in the figure on page 6 of notes is for recombination lifetime limited by shallow donors, calculate the capture cross-section for the shallow donors.

(c) Assuming that lower line on plot gives lifetime due to Auger recombination, determine \( K_n \) for Auger recombination in this material.

(d) How does the effective lifetime change as the injection level increases into high level injection? Calculate and plot \( \tau \) versus \( \Delta p = \Delta n \) for doping of \( 10^{16} \text{cm}^{-3} \) and \( 10^{19} \text{cm}^{-3} \).

2. A silicon membrane with thickness 5 \( \mu \text{m} \) has a uniform donor concentration of \( 5 \times 10^{17} \text{cm}^{-3} \). One end is irradiated and hole-electron pairs are generated at a rate of \( 10^{18} \text{cm}^{-2} \text{s}^{-1} \) at the surface. At the other surface, the recombination velocity is \( 10^4 \text{cm/s} \). Assume \( D_p = 4 \text{cm}^2/\text{s} \) and \( \tau_p = 100\text{ns} \). Ignore recombination at the irradiated end.

(a) Calculate and sketch the concentrations of holes and electrons as a function of distance. Use the Diffusion Approximation (drift current is negligible for minority carriers under low level injection) and the Quasi-Neutrality Approximation (\( \Delta n \approx \Delta p \)).

(b) What percentage of the injected carriers recombine at the surface?

(c) Based on your solution to (a), calculate the majority carrier diffusion current as function of position.

(d) Since \( J_n = J_p \) in steady state (carriers are generated and recombine as pairs) for this system, determine the majority carrier drift current and from this the electric field.

(e) Use your calculation of the electric field to test the Diffusion Approximation.

(f) Use your calculation of the electric field and Poisson’s equation to determine the charge density and thus test the Quasi-Neutrality Approximation.

3. Consider silicon heavily doped with arsenic at a concentration of \( 4 \times 10^{19} \text{cm}^{-3} \) such that an impurity band with density of states

\[
N_{ib}(E) = \begin{cases} 
\frac{4\pi \times 10^{19} \text{cm}^{-3}}{0.12 \text{eV}} \cos \left[ \frac{\pi(E - E_c + 0.03 \text{eV})}{0.12 \text{eV}} \right] & E_c + 0.03 \text{eV} > E > E_c - 0.09 \text{eV} \\
0 & \text{otherwise}
\end{cases}
\]

is formed. Assume that the conduction and valence band density of states, as well as \( E_g = E_c - E_v \) are unchanged.

(a) If the electron concentration is equal to the donor doping, what would be the Fermi level location at 0K (absolute zero)? (Hint: Note simple form of Fermi Dirac statistics at 0K.)

(b) If the Fermi level remains in the same location as the temperature rises (as it will approximately in degenerate material), what would be the hole concentration at room temperature (300K)? How does the \( pn \) product \( (n_{ie}^2) \) compare to \( n_i^2 \) in lightly-doped material (Assume all electrons in overlapping impurity/conduction band are mobile)?