Exam #2 Solutions

EE 531
Winter '05

1 $S = 75\,mV/decade = m * \frac{kT}{q} \ln 10 = m\left(60 \frac{mV}{decade}\right)$

$m = 1.25 = 1 + \frac{C_d'}{C_{ox}'}$ $\quad C_{ox}' = \frac{K_{ox}\epsilon_0}{t_{ox}} = 1.73\times10^{-6}\frac{F}{cm^2}$, $\quad C_d' = \frac{K_s\epsilon_0}{W_{dm}}$

$W_{dm} = 4\frac{K_s}{K_{ox}} t_{ox} = 12\, t_{ox} = 24\,nm$

$V_T = V_{FB} - 2\psi_F - \frac{Q_{d,max}'}{C_{ox}'} = -0.4V$

$V_{FB} = \phi_{MS} - \cancel{\frac{Q_{ss}'}{C_{ox}'}}^{\text{neglect}} = \phi_{MS} = \cancel{\chi_s} + \frac{E_g}{q} - \left(\cancel{\chi_s} + \frac{kT}{q}\ln\frac{N_c}{N_d(W_{dm})}\right)$

$\psi_F = \frac{kT}{q}\ln\frac{N_d(W_{dm})}{n_i}$ $\qquad \phi_{MS} - 2\psi_F = \frac{kT}{q}\ln\frac{N_v}{N_d(W_{dm})}$

Assume $N_d(W_{dm}) = 10^{19}\ cm^{-3}$ $\qquad = 0.024$

$V_T = 0.024 - \frac{Q_{d,max}}{C_{ox}'} = -0.4V$

$\frac{Q_{d,max}}{q} = \frac{C_{ox}'}{q}(0.424V) = 4.58\times10^{12}\ cm^{-2}$

$N_d^{avg} = Q_{d,max}/W_{dm} = 1.91\times10^{18}\ cm^{-3}$

$\psi_s = -2\psi_F = -1.07V = -\frac{q}{K_s\epsilon_0}\int_0^{W_{dm}} x\, N_d(x)\,dx = \frac{q\, W_{dm}}{K_s\epsilon_0} N_d^{avg}\, x_{avg}$

$x_{avg} = 15.1\,nm > \frac{W_{dm}}{2}$, so doping is heavier away from surface, consistent w/ $N_d^{avg} < N_d(W_{dm})$

Assume $N_d(x) = \begin{cases} N_1 & x < a \\ N_2 = 10^{19} & x > a \end{cases}$

$N_1 a + N_2(24nm - a) = 4.58\times10^{12}\ cm^{-2}$

$N_1 a^2/2 + [a + (24nm - a)/2]\, N_2 (24nm - a) = 15.1nm \times 4.58\times10^{12}\ cm^{-2}$

$\Rightarrow a = 22.54,\ N_1 = 1.38\times10^{18}\ cm^{-3}$

$$N_1 a + (24-a) = 4.58 \Rightarrow N_1 = \frac{a-19.42}{a}$$

$$N_1 a^2 + (a+24)(24-a) = 2 \times 15.1 \times 4.58 = 138.3$$

$$(a-19.42)(a) + 24^2 - a^2 = 138.3$$

$$\cancel{a^2} - 19.42a + 24^2 - \cancel{a^2} = 138.3 \Rightarrow a = \frac{576-138.3}{19.42} = 22.54\,\mu m$$

$$N_1 = \frac{22.54-19.42}{22.54} = 0.138 \times 10^{19}\,cm^{-3}$$

$$= 1.38 \times 10^{18}\,cm^{-3}$$

Uniformly-doped structure has $Q_{d,max} = \sqrt{2K_s\epsilon_0 q N_d |2\psi_F|}$

Assume $N_d \sim 3\times10^{18}\ \text{cm}^{-3}$ $\psi_F \sim 0.50$

$$V_T = 0.055 - \frac{\sqrt{2K_s\epsilon_0 q N_d |2\psi_F|}}{C_{ox}'} = -0.4$$

$$N_d = \frac{\left[(C_{ox}')(0.455V)\right]^2}{2K_s\epsilon_0 q(1V)} = 1.87\times10^{18}\ \text{cm}^{-3}$$

$$W_{dm} = \sqrt{\frac{2K_s\epsilon_0 |2\psi_F|}{q N_d}} = 26.3\ \text{nm}$$ (greater than for retrograde profile)

Since W_{dm} is greater for uniform doping, m is smaller

(a) Uniform doping => steeper subthreshold slope (m smaller)

(b) Uniform doping => Higher V_{DS}^{sat} (m smaller)

(c) If doping in retrograde structure remains high under drain, it will have larger C_j, but if only a narrow heavily-doped region is used it can have lower C_j

(d) Retrograde => Smaller W_{dm}, so less charge sharing w/ source & drain.

2. $V_T^{LC} = V_{FB} + 2\psi_B + \frac{\sqrt{4\kappa_s\epsilon_0 q N_a 2\psi_B}}{C_{ox}'} = 0.5V$

$C_{ox}' = \frac{\kappa_{ox}\epsilon_0}{t_{ox}} = 1.38\times10^{-6}\frac{F}{cm^2}$ $\quad W_{dm} = 26.3nm$

$V_{FB} + 2\psi_B = -\frac{E_g}{2q} - \psi_B + 2\psi_B = -0.56V + \psi_B$

$-0.56V + \psi_B + 0.59\sqrt{\frac{N_a\psi_B}{10^{18}}} = 0.5$

$N_a = \left[\frac{(1.06-\psi_B)}{0.59}\right]^2 \frac{10^{18}}{\psi_B}$ $\quad \psi_B = \frac{kT}{q}\ln\frac{N_a}{n_i}$

$\Rightarrow N_a = 1.85\times10^{18}\ cm^{-3}$, $\psi_B = 0.495V$

$\frac{I_{off}}{W} = \mu_{eff}\frac{1}{L}\left(\frac{kT}{q}\right)^2 C_{ox}'(m-1)e^{q[0-V_t(V_{ds},L)]/mkT} = 10^{-4}\frac{A}{\mu m}$

$= 7.9\times10^{-8}\frac{1}{L}e^{-V_t/mkT} = 1 A/cm$

$m = 1 + \frac{3t_{ox}}{W_{dm}} = 1 + \frac{3\times2.5nm}{26.3nm} = 1.285$

$\mathcal{E}_{eff} = \left[\frac{V_t+0.2}{3t_{ox}} + \frac{\overset{0}{V_g}-V_t}{6t_{ox}}\right] = 0.6\frac{MV}{cm} \Rightarrow \mu_{eff} \cong 300\frac{cm^2}{Vs}$

$V_t(V_{ds}=1.5V, L) = 0.5V - \overbrace{8(m-1)\sqrt{\psi_{bi}(\psi_{bi}+V_{ds})}}^{3.76}\, e^{-\pi L/2mW_{dm}}$

$\psi_{bi} \cong \psi_B + \frac{E_g}{2q} = 1.06V$

$L = 100nm \Rightarrow \Delta V_T = 0.036V \Rightarrow \quad = 6.9\times10^{-9}A/cm$

$L = 50nm \Rightarrow \Delta V_T = 0.36V \Rightarrow \frac{I_{off}}{W} =$

$\underline{L = 95nm} \Rightarrow \Delta V_T = 0.045V \Rightarrow \frac{}{} = 0.97\times10^{-0} A/cm = 10^{-4}\frac{A}{\mu m}$

3 (a) $\frac{1}{C_g} = \frac{1}{C_{ox}} + \frac{1}{C_{poly}} + \frac{1}{C_s}$ $\qquad \psi_B = \frac{kT}{q} \ln \frac{N_a}{n_i} = 0.52\,V$

$$Q'_{d,max} = \sqrt{2 K_s \epsilon_0 q N_a |2\psi_B|} = 1.31 \times 10^{-6}\,C/cm^2$$

$$Q^* = Q_d + \frac{11}{32} Q_i = \left[13.1 + \frac{11}{32} 8\right] \times 10^{-7} \frac{C}{cm^2} = 1.585 \times 10^{-6} \frac{C}{cm^2}$$

$= 1.59 \times 10^{-2}\,C/m^2$

$8.854 \times 10^{-12}\,F/m$

$$x_{av}^{QM} = \left(\frac{9 K_s \epsilon_0 h^2}{16 \pi^2 m_x q Q^*}\right)^{1/3}$$

$= 1.1 \times 10^{-9}\,m = 1.1\,nm$

$$\mathcal{E}_s = \frac{Q_d + Q_i}{K_s \epsilon_0} = 2.04 \times 10^6 \frac{V}{cm},$$

so dominated by 1st sub-band which has $m_x = m_e = 0.92 m_0$

$$C'_s \equiv C'_i \cong \frac{K_s \epsilon_0}{x_{av}^{QM}} = 9.4 \times 10^{-6} \frac{F}{cm^2}$$

$$C'_{ox} = \frac{K_{ox} \epsilon_0}{t_{ox}} = 2.3 \times 10^{-6}\,F/cm^2$$

$$C'_{poly} = \frac{K_s \epsilon_0}{x_{poly}} = 3.9 \times 10^{-6}\,F/cm^2$$

$$x_{poly} = \frac{Q_s}{q N_{poly}} = \frac{21.1 \times 10^{-7}\,C/cm^2}{q (5 \times 10^{19}\,cm^{-3})}$$

$= 2.63 \times 10^{-7}\,cm$

$= 2.63\,nm$

$$C_g = \left(\frac{1}{9.4} + \frac{1}{2.3} + \frac{1}{3.9}\right)^{-1} \mu F/cm^2$$

$= 1.25 \times 10^{-6}\,F/cm^2$

(b) $V_{GS} = V_{GB} = V_{FB} - \frac{Q_s'}{C_{ox}'} + \psi_s + V_{poly}$ $\qquad 0.19\, m_0$

$\psi_s = 2\psi_B - \Delta\psi_s^{QM}$
$= -1.04 - 0.21 = -1.25\,V$

$\psi_B = -\frac{kT}{q}\ln\left(5\times10^{8}\right) = -0.52\,V$

$\Delta\psi_s^{QM} \cong \frac{E_0}{q} - \underbrace{\frac{kT}{q}\ln\left(\frac{8\pi q m_d \mathcal{E}_s}{h^2 N_v}\right)}_{-0.066\,V}$ $\qquad 2.04\times10^{8}$ $\qquad 2.5\times10^{25}$

$E_0 = \left[\frac{3hq\mathcal{E}_s}{4\sqrt{2m_x}}\left(3/4\right)\right]^{2/3}$
$= 0.278\,eV$

$V_{poly} = -\frac{Q_s}{2C_{poly}}$
$= -\frac{2.11\times10^{-6}\,C/cm^2}{2(3.9\times10^{-6}\,F/cm^2)}$
$= -0.27\,V$

$V_{ox} = -\frac{Q_s'}{C_{ox}'} = -\frac{2.11\times10^{-6}\,C/cm^2}{2.3\times10^{-6}\,F/cm^2} = -0.92\,V$

$V_{GS} = 1\,V - 0.92\,V - 1.25\,V - 0.27\,V = \underline{-1.44\,V}$