Homework #5 - EE 531

due 5/3/11

1. The scattering rate due to ionized impurity scattering in a single parabolic and spherically symmetric band is:

$$S(k,k') = \frac{2\pi}{\hbar}\frac{N_I Z^2 q^4}{\Omega\epsilon_s^2}\frac{\delta(E_{k'}-E_k)}{[|k-k'|^2+q_D^2]^2} = \frac{2\pi}{\hbar}\frac{N_I Z^2 q^4}{\Omega\epsilon_s^2}\frac{\delta(E_{k'}-E_k)}{[2k^2(1-\cos\theta)+q_D^2]^2},$$

where k and k' are measured relative to the bottom of the conduction band. N_I is the ionized impurity concentration, θ is the angle between k and k', and q_D is the inverse of the Debye length. Via integration, the rate of scattering through a given angle θ is

$$P(\theta,k) = \frac{\pi N_I Z^2 q^4 N(E_k)}{\hbar\epsilon_s^2}\frac{\sin\theta}{[2k^2(1-\cos\theta)+q_D^2]^2},$$

where $N(E_k)$ is the density of states within the band. It is possible to obtain the rate of momentum loss by integrating $P(\theta,k)(1-\cos\theta)$ over possible values of θ (0 to π). The result is:

$$\frac{1}{\tau_m} = \frac{\pi N_I Z^2 q^4 N(E_k)}{\hbar\epsilon_s^2 k^4}\left[\ln\left|1+\frac{4k^2}{q_D^2}\right| - \frac{4k^2/q_D^2}{1+4k^2/q_D^2}\right].$$

   (a) Plot the mobility as limited by ionized impurity scattering versus temperature for doping of 10^{16}cm^{-3} and 10^{18}cm^{-3} in silicon. Rather than integrating over all possible k values, you can instead use an average value of k for the given temperature ($KE = 3k_BT/2$). Assume the scattering is within a single minima which can be approximated to be spherically symmetric with a single effective mass $(m_l m_t^2)^{1/3}$. What is the approximate power law dependence?

   (b) Plot the mobility versus doping at room temperature. Comment on your results.

2. Consider a spatially uniform system with no generation/recombination. Using the hydrodynamic equations (see notes):

   (a) Determine an expression for the dependence of the average electron energy on electric field in steady-state if it is given that the momentum relaxation time depends inversely on the energy as $\tau_m = AE^{-3/2}$ and the steady-state drift velocity depends on the electric field as $|v_d| = v_{\text{sat}}\mathcal{E}/(\mathcal{E}+\mathcal{E}_c)$, where $\mathcal{E}$ is the electric field.

   (b) What must be the dependence of the energy relaxation time (τ_E) on average electron energy? Sketch your result.

3. A MOS capacitor is made with a silicon substrate doped with $N_a = 5\times10^{17}\text{cm}^{-3}$ of boron, 6 nm of silicon dioxide, and an n^+ polysilicon gate doped such that $E_f - E_c = 0.05\,\text{eV}$. $Q'_{ss}/q = 5\times10^{10}\,\text{cm}^{-2}$. Assuming that inversion and accumulation charges approximate a sheet of charge at the interface and that weak inversion charges can be neglected, determine the charge on the gate, the voltage dropped across the oxide and the voltage dropped across the silicon with the following voltages applied between the gate and the substrate:

   (a) $V_{gb} = -1\,\text{V}$

   (b) $V_{gb} = 0.0\,\text{V}$

   (c) $V_{gb} = 2\,\text{V}$

   Sketch the charge densities, electric fields and energy band diagrams in each case. What are the capacitances at low and high frequencies in each of the above cases?

4. Problem 2.6 in Text (Taur and Ning).

5. Sentaurus Device problem to be added later.