EE 531
Winter 2017

Homework #5 Solutions

1. $\mu = \dfrac{q\tau_m}{m^*}$

$$\frac{1}{\tau_m} = \frac{\pi N_I Z^2 q^4 N(E_k)}{\hbar \epsilon_s^2 k^4}\left[\ln\left|1+\frac{4k^2}{h_D^2}\right| - \frac{4k^2/h_D^2}{1+4k^2/h_D^2}\right]$$

$$N(E_k) = \frac{4\pi}{h^3}(2m^*)^{3/2}E_k^{1/2}, \text{ where } m^* = (m_\ell m_t^2)^{1/3} = 0.33m_0 \text{ and } E_k = E - E_c$$

$$h_D^2 = \frac{q^2 n_0}{\epsilon_s kT} \qquad k^2 = \frac{2E_k m^*}{\hbar^2} \qquad Z = 1 \text{ (singly ionized)}$$

a) Assume $n_0 = N_I$, and that on average $E_k = \frac{3}{2}kT$

Substituting above, $$\mu = \frac{q\hbar\epsilon_s^2 k^4 h^3}{m^*\pi N_I Z^2 q^4 4\pi(2m^*)^{3/2}E_k^{1/2}\left[\ln|1+\beta| - \frac{\beta}{1+\beta}\right]}$$

where $\beta = \dfrac{4k}{h_D}$

$$\beta = \frac{4(2E_k m^*)\epsilon_s(kT)}{\hbar^2 q^2 N_I}$$

$$= \frac{\cancel{\hbar}\epsilon_s^2\left(\frac{4E_k^2 m^{*2}}{\hbar^{4}}\right)h^3}{m^* N_I q^3(2\pi)^2(2m^*)^{3/2}E_k^{1/2}[\quad]}$$

$$\mu = \frac{\epsilon_s^2 4E_k^{3/2}(2\pi)}{m^{*1/2}N_I q^3(2)^{3/2}[\quad]}$$

$$\mu = \frac{\epsilon_s^2\cancel{4}(3kT)^{3/2}(2\pi)}{m^* N_I q^3 2^3[\quad]} = \frac{\epsilon_s^2 3^{3/2}(kT)^{3/2}\pi}{m^* N_I q^3[\quad]}$$

$(1J)^{1/2} = \left(\frac{kg\, m^2}{s^2}\right)^{1/2}$

$$\frac{C^{1/2}}{V^{1/2}}\frac{cm}{kg^{1/2}} = \frac{C^{1/2}V^{1/2}cm}{kg^{1/2}V}$$

At $T = 300K$, $N_I = 10^{16}\ cm^{-3}$: ← note, we used $\frac{kT}{q}$

$$\mu = \frac{12^2(8.854\times10^{-14}F/cm)^2(3(0.026\ V))^{3/2}\pi}{(0.33\times9.11\times10^{-31}kg)^{1/2}(10^{16}cm^{-3})(1.6\times10^{-19}C)^{3/2}[\quad]} = \frac{220}{[\quad]}\ \frac{\frac{C^2}{V^2 cm^2}V^{3/2}}{kg^{1/2}cm^{-3}C^{3/2}}$$

$$= 2.2\times10^4\ \frac{cm^2}{V\cdot s}$$

2 (a) From Eq (18)

$$\cancel{\frac{d\langle p\rangle}{dt}} = -\frac{1}{m^*}\cancel{(\langle p\rangle\cdot\nabla_r)\langle\vec{p}\rangle} - q\mathcal{E} - \cancel{\frac{1}{n}\nabla_r(nkT_e)} - \frac{\langle\vec{p}\rangle}{\langle\tau_m\rangle}$$

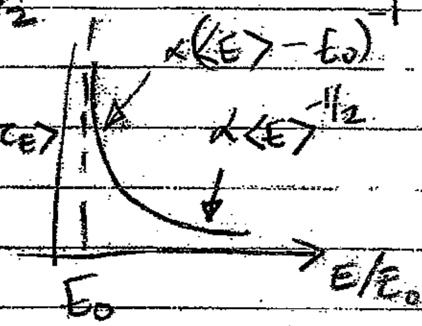
Steady-state   Spatially uniform

$$\langle\vec{p}\rangle = -q\mathcal{E}\langle\tau_m\rangle = -q\mathcal{E}\left[A\langle E\rangle^{-3/2}\right]$$

$$|v_d| = \left|\frac{\langle p\rangle}{m^*}\right| = \frac{v_{dsat}\cancel{\mathcal{E}}}{\mathcal{E}+\mathcal{E}_c} = \frac{q\cancel{\mathcal{E}}A\langle E\rangle^{-3/2}}{m^*}$$

$$\langle E\rangle^{-3/2} = \frac{v_{dsat}\,m^*}{qA(\mathcal{E}+\mathcal{E}_c)}$$

$$\langle E\rangle = \left(\frac{qA(\mathcal{E}+\mathcal{E}_c)}{v_{dsat}\,m^*}\right)^{2/3}$$



(b) Use Eq. (19) to get $0 = -\frac{1}{m^*}q\mathcal{E}\cdot\langle p\rangle - \frac{\langle E\rangle - E_0}{\langle\tau_E\rangle}$

$$+q\mathcal{E}\left(\frac{q\mathcal{E}}{m^*}A\langle E\rangle^{-3/2}\right) = \frac{\langle E\rangle - E_0}{\langle\tau_E\rangle}$$

$$\langle\tau_E\rangle = \frac{(\langle E\rangle - E_0)\,m^*\langle E\rangle^{3/2}}{q^2\mathcal{E}^2A} = \frac{(\langle E\rangle - E_0)\,m^*\langle E\rangle^{3/2}}{A\left(\frac{v_{dsat}\,m^*}{A}\langle E\rangle^{3/2} - q\mathcal{E}_c\right)^2}$$

From above,

$$\mathcal{E} = \frac{v_{dsat}\,m^*}{qA}\langle E\rangle^{3/2} - \mathcal{E}_c$$

For $\mathcal{E} = 0$, $E = E_0$, so $E_0 = \left(\frac{qA\mathcal{E}_c}{v_{dsat}\,m^*}\right)^{2/3}$; $\mathcal{E}_c = \frac{v_{dsat}\,m^*}{qA}E_0^{3/2}$

$$\langle\tau_E\rangle = \frac{(\langle E\rangle - E_0)\,m^*\langle E\rangle^{3/2}}{\frac{(v_{dsat}\,m^*)^2}{A}\left(\langle E\rangle^{3/2} - E_0^{3/2}\right)^2}$$

See sketch above

$$\beta = \frac{4\left(3kT\,m^*\right)\epsilon_s kT}{\hbar^2 q^2 N_I} = \frac{12\left(kT/q\right)^2 m^* \epsilon_s}{\hbar^2 N_I}$$

$$= \frac{12\left(0.026\ V\right)^2\left(0.33\times 9.11\times 10^{-31}\ kg\right)\left(8.854\times 10^{-14}\ \frac{F}{cm}\right)12}{\left(1.05\times 10^{-34}\ J\cdot s\right)^2\left(10^{16}\ cm^{-3}\right)} \quad (1eV = 1J)$$

$$= 2.4\times 10^{7}\ \frac{V^2\,kg\,\frac{C}{V\,cm}}{J^2 s^2\,cm^{-3}} = 2.4\times 10^{7}\ \frac{\cancel{kg}}{\cancel{kg}\,m^2\,cm^{-2}} = 2400$$

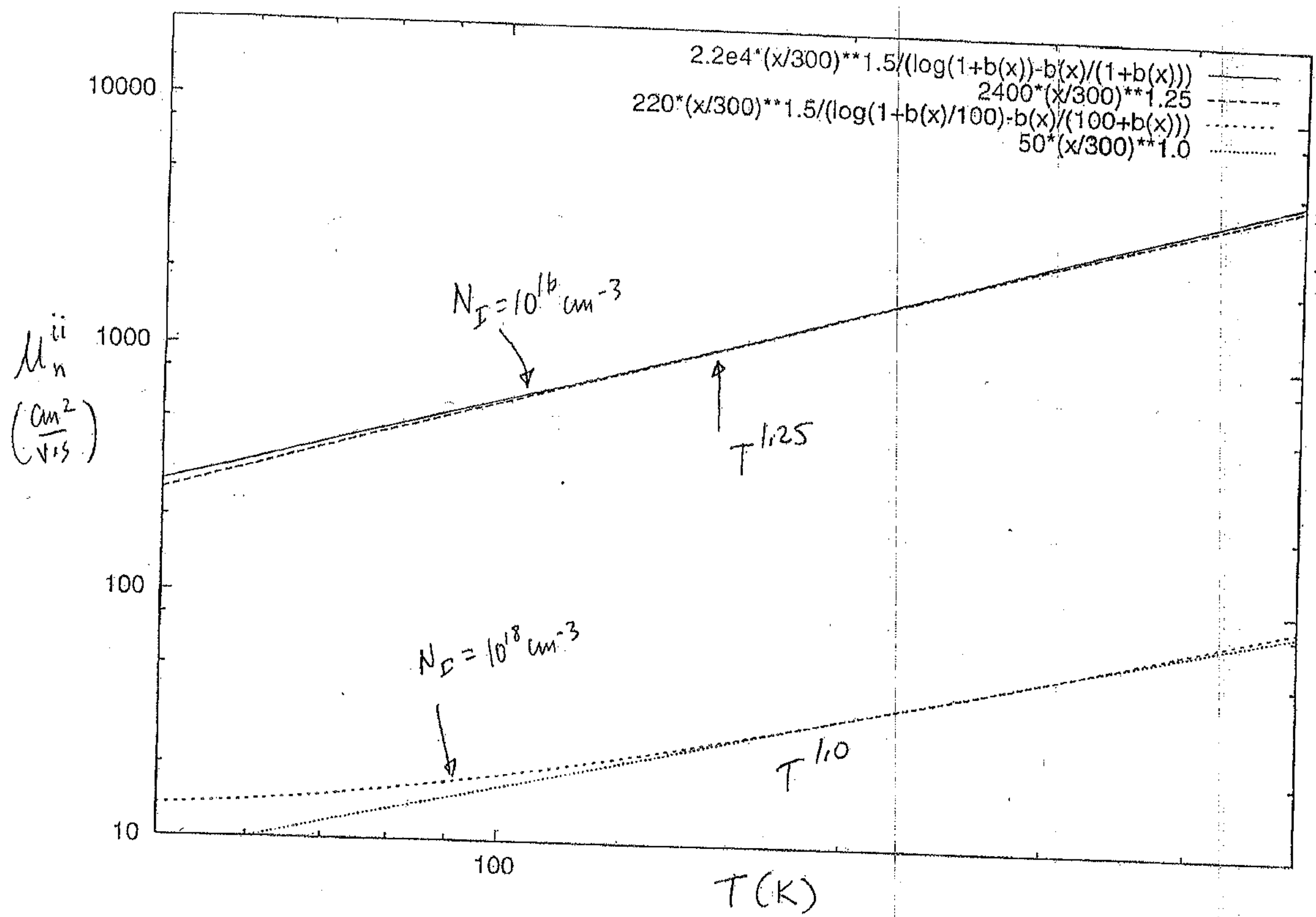
$$\beta = 2400\left[\frac{\left(T/300K\right)^2}{\left(N_I/10^{16}\,cm^{-3}\right)}\right] \qquad \mu = \frac{2.2\times 10^{4}\left(T/300K\right)^{3/2}}{\left(N_I/10^{16}\,cm^{-3}\right)\left[\ln|1+\beta| - \frac{\beta}{1+\beta}\right]}$$

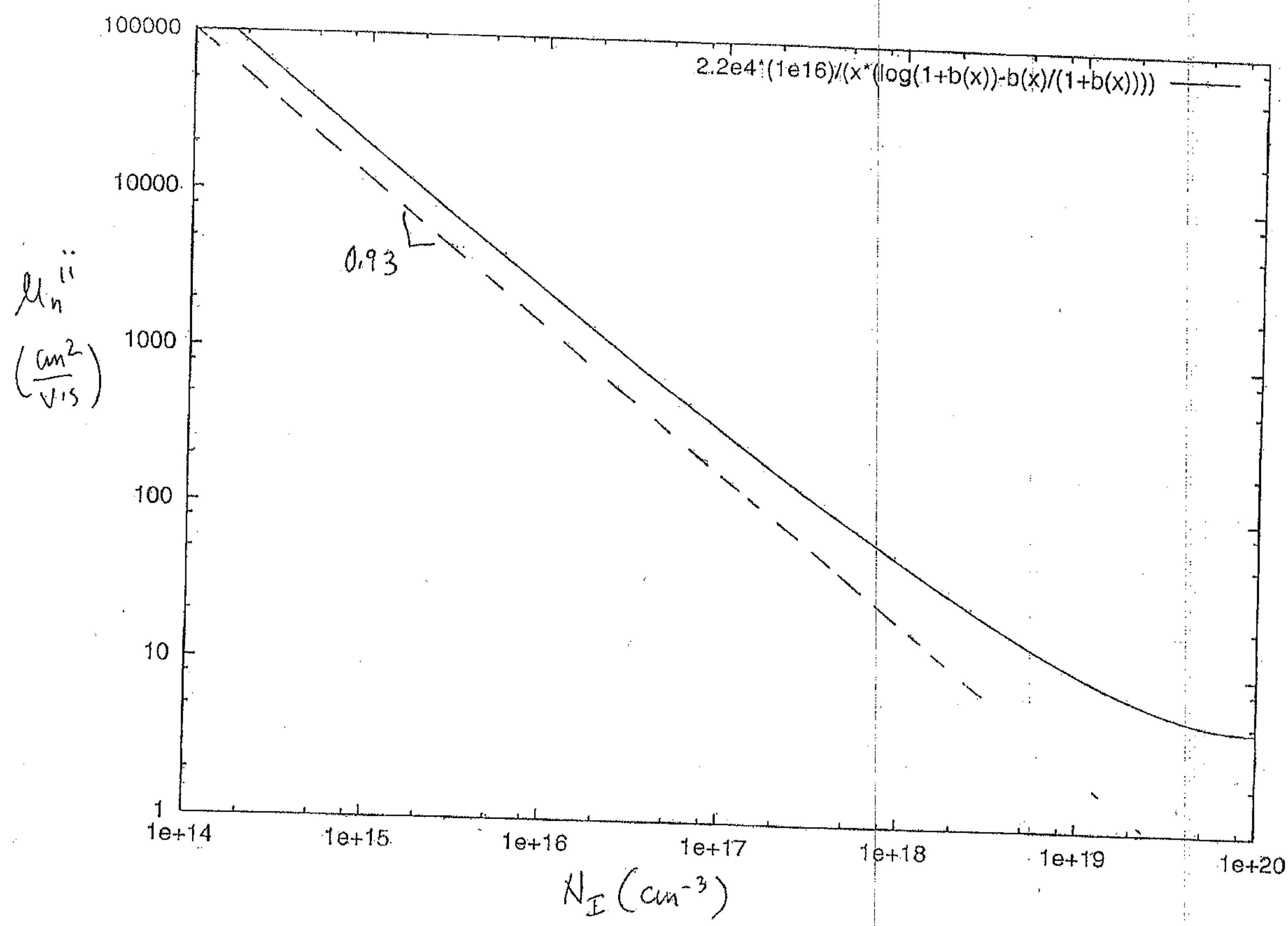
Looking at plot on next page; near 300K, $\mu \propto T^{1.25}$ for $N_I = 10^{16}$
$\mu \propto T$ for $N_I = 10^{18}$

As expected, mobility as limited by ionized impurity scattering increases with T due to higher speed of electrons. The power dependence varies with T and doping level. The dependence is slightly weaker than the simple $T^{3/2}$ dependence predicted by a simple model (leading term).

(b) Plot is shown on following page based on $\beta(T,N_I)$, $\mu(T,N_I)$ above. As expected, mobility drops with increased doping. The power dependence is slightly weaker than linear at low N_I, and saturates at high N_I.

Note that at low N_I or high T, μ will be limited by phonon rather than ionized impurity scattering.


2.2e4*(x/300)**1.5/(log(1+b(x))-b(x)/(1+b(x)))
2400*(x/300)**1.25
220*(x/300)**1.5/(log(1+b(x)/100)-b(x)/(100+b(x)))
50*(x/300)**1.0
10000
1000
100
10
100
μ_n^{ii} $\left(\frac{cm^2}{V \cdot s}\right)$
$N_I = 10^{16}\ cm^{-3}$
$T^{1.25}$
$N_I = 10^{18}\ cm^{-3}$
$T^{1.0}$
T (K)


2.2e4*(1e16)/(x*(log(1+b(x))-b(x)/(1+b(x))))
100000
10000
1000
100
10
1
1e+14
1e+15
1e+16
1e+17
1e+18
1e+19
1e+20
0.93
μ_n^{ii} $\left(\frac{cm^2}{V \cdot s}\right)$
N_I (cm^{-3})

$$C_{ox}' = \frac{K_{ox}\epsilon_o}{x_{ox}} = \frac{(3.9)(8.854\times10^{-14}\,F/cm)}{60\times10^{-8}\,cm} = 5.76\times10^{-7}\,F/cm^2$$

3. $$\phi_{ms} = \frac{1}{q}(E_f - E_c)_s - \frac{1}{q}(E_f - E_c)_m$$

$$= -\frac{E_g}{q} + \frac{kT}{q}\ln\left(\frac{N_v}{N_a}\right) - 0.05\ V = -1.12eV + kT\ln\left(\frac{1.8\times10^{19}}{5\times10^{17}}\right) - 0.05$$

$$= -1.08\,V$$

$$V_{FB} = \phi_{ms} - \frac{Q_{ss}'}{C_{ox}'} = -1.08\,V - \frac{(1.6\times10^{-19}e)(5\times10^{10}\,cm^{-2})}{5.76\times10^{-7}\,F/cm^2} = -1.09\,V$$

(a) $V_{gb} = -1V > V_{FB} = -1.09V \Rightarrow$ depletion

$$V_{gb} = \phi_{ms} + \psi_{ox} + \psi_s = V_{FB} + \Delta\psi_{ox} + \psi_s$$

$$= -1.09V - \frac{Q_s'}{C_{ox}'} + \psi_s$$

$$Q_s' = -\sqrt{2qN_aK_s\epsilon_o\psi_s} = -4.07\times10^{-7}\frac{C}{V^{1/2}cm^2}\sqrt{\psi_s} = -4.5\times10^{-8}\,C/cm^2$$

$$V_{gb} = -1.09V + 0.71\sqrt{\psi_s} + \psi_s = -1V \Rightarrow \underline{\underline{\psi_s = 0.012V}}$$

$$Q_g = -(Q_s' + Q_{ss}') = -\left(-4.5\times10^{-8}\,C/cm^2 + 8\times10^{-9}\,C/cm^2\right)$$

$$= \underline{\underline{3.7\times10^{-8}\,C/cm^2}}$$

$$\Delta\psi_{ox} = 0.71\sqrt{\psi_s} = 0.08\,V \quad \psi_{ox} = -\frac{Q_{ss}'}{C_{ox}'} + \Delta\psi_{ox} = \underline{\underline{0.07\,V}}$$

$$\psi_B = \frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) = 0.45\,V$$

(b) $$V_T = V_{FB} - \frac{Q_d'^{max}}{C_{ox}'} + 2\psi_B$$

$$Q_d'^{max} = -4.07\times10^{-7}\frac{C}{V^{1/2}cm^2}\sqrt{\psi_s} = -C_{ox}'\,0.71\sqrt{0.9V}\,V^{1/2}$$

$$V_T = -1.09V + 0.71\sqrt{0.9} + 0.9 = 0.48V$$

$V_{gb} = 0V < V_T \Rightarrow$ still depletion

$V_{gb} = -1.09 + 0.71\sqrt{\psi_s} + \psi_s = 0 \qquad \underline{\underline{\psi_s = 0.56V}}$

$Q_g' = -(Q_s' + Q_{ss}') = \underline{\underline{3.0 \times 10^{-7}\ C/cm^2}}$

$\psi_{ox} = V_{gb} - \phi_{ms} - \psi_s = 0 + 1.08 - 0.56 = \underline{\underline{0.52V}}$

(c) $V_{gb} = 2V > V_T \Rightarrow$ inversion

Neglecting any voltage required to form inversion layer once $n_s = p_{bulk}$:

$$\psi_s = 2\psi_B = 0.9V$$

$\psi_{ox} = V_{gb} - \phi_{ms} - \psi_s = 2.0 + 1.08 - 0.9 = 2.18V$

$Q_g' = C_{ox}'\psi_{ox} = 1.26 \times 10^{-6}\ C/cm^2$

More accurately, in inversion $\psi_s \cong 2\psi_B + 6kT/q = 1.06V$

Then $\psi_{ox} = 2.02V$ and $Q_g' = 1.16 \times 10^{-6}\ C/cm^2$

It is also possible to use result from text that

$$Q_s = -\sqrt{2qK_s\epsilon_0 N_a}\left(\psi_s + \frac{kT}{q}\frac{n_i^2}{N_a^2}e^{q\psi_s/kT}\right)^{1/2}$$

$$= -4.07 \times 10^{-7}\frac{C}{cm^2 V^{-1/2}}\left[\psi_s + 2 \times 10^{-17} V e^{q\psi_s/kT}\right]^{1/2}$$

$V_{gb} = V_{FB} + 0.71\left[\psi_s + 2 \times 10^{-17} e^{q\psi_s/kT}\right]^{1/2} + \psi_s = 2V \Rightarrow \psi_s = 1.05V$

Very close to 2nd estimate

4 T&N 2.6 (a) $\frac{d}{d\psi_s}\int_0^{\psi_s} f(\psi)\,d\psi = f(\psi_s)$

$$C_d' = -\frac{dQ_d'}{d\psi_s} = qN_a \frac{\left(1-e^{-q\psi_s/kT}\right)}{\left\{\left(\frac{2kTN_a}{K_s\epsilon_o}\right)\left[\left(e^{-q\psi_s/kT}+\frac{q\psi_s}{kT}-1\right)+\frac{n_i^2}{N_a^2}\left(e^{\frac{q\psi_s}{kT}}-\frac{q\psi_s}{kT}-1\right)\right]\right\}^{1/2}}$$

$$= q\left(\frac{K_s\epsilon_o N_a}{2kT}\right)^{1/2} \frac{\left(1-e^{-q\psi_s/kT}\right)}{[\quad]^{1/2}}$$

$$C_i' = -\frac{dQ_i'}{d\psi_s} = \frac{n_i^2}{N_a^2} q\left(\frac{K_s\epsilon_o N_a}{2kT}\right)^{1/2} \frac{\left(e^{q\psi_s/kT}-1\right)}{[\quad]^{1/2}}$$

(b) $$C_{sc}' = -\frac{dQ_s}{d\psi_s} = -\frac{d}{d\psi_s}\left\{(2K_s\epsilon_o kTN_a)\left[\left(e^{-q\psi_s/kT}+\frac{q\psi_s}{kT}-1\right)+\frac{n_i^2}{N_a^2}\left(e^{\frac{q\psi_s}{kT}}-\frac{q\psi_s}{kT}-1\right)\right]\right\}^{1/2}$$

$$= -\frac{1}{2}(2K_s\epsilon_o kTN_a)^{1/2} \frac{\left[\left(-\frac{q}{kT}e^{-q\psi_s/kT}+\frac{q}{kT}\right)+\frac{n_i^2}{N_a^2}\left(\frac{q}{kT}e^{q\psi_s/kT}-\frac{q}{kT}\right)\right]}{[\quad]^{1/2}}$$

same as above

$$= \left(\frac{K_s\epsilon_o kTN_a}{2}\right)^{1/2}\left(\frac{q}{kT}\right) \frac{\left(1-e^{-q\psi_s/kT}\right)+\frac{n_i^2}{N_a^2}\left(e^{q\psi_s/kT}-1\right)}{[\quad]^{1/2}}$$

$$= \left(\frac{K_s\epsilon_o N_a}{2kT}\right)^{1/2} q \frac{\left(1-e^{-q\psi_s/kT}\right)+n_i^2/N_a^2\left(e^{q\psi_s/kT}-1\right)}{[\quad]^{1/2}}$$

$$= C_d' + C_i'$$

(c) For $\psi_s \cong 2\psi_B$: $C_d' \cong q\left(\frac{K_s\epsilon_o N_a}{2kT}\right)^{1/2} \frac{1}{\left[\frac{q\psi_s}{kT}+\cancel{\frac{n_i^2}{N_a^2}\left(e^{2q\psi_B/kT}\right)}\right]^{1/2}}$ $\sim 1 \ll \frac{q\psi_s}{kT}$

$\cong \left(\frac{N_a}{n_i}\right)^2$

$$\cong q\left(\frac{K_s\epsilon_o N_a}{2kT}\right)^{1/2} \frac{1}{\left[\frac{q\psi_s}{kT}\right]^{1/2}}$$

$$C_i' \cong q\left(\frac{K_s\epsilon_0 N_a}{2kT}\right)\frac{n_i^2}{N_a^2}\frac{e^{2q\psi_B/kT}}{\left[\frac{q\psi_s}{kT}\right]^{1/2}} \cong C_d'$$

$$e^{2q\psi_B/kT} \cong \left(\frac{N_a}{n_i}\right)^2$$

(d) For $\psi_s \gtrsim 2\psi_B$

$$C_d' \cong q\left(\frac{K_s\epsilon_0 N_a}{2kT}\right)^{1/2}\frac{1}{\left[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_a^2}e^{q\psi_s/kT}\right]^{1/2}}$$

$$\cong \left(\frac{qN_a K_s\epsilon_0}{2\psi_s}\right)^{1/2}\frac{1}{\left[1 + \frac{kT}{q\psi_s}e^{q(\psi_s-2\psi_B)/kT}\right]^{1/2}}$$

For $\psi_s \lesssim 2\psi_B$, $\frac{kT}{q\psi_s}e^{q(\psi_s-2\psi_B)/kT} \ll 1$

and we get our usual result ignoring any voltage drop across inversion layer (δ-function approx) However, as the inversion layer builds up, the voltage drop becomes significant. For $\psi_s - 2\psi_B \gtrsim 2kT/q$ the second term starts to dominate and the dependence of Q_d' on ψ_s weakens.

Under typical operation (strong inversion) $\psi_s \sim 2\psi_B + 6kT/q$, while Q_d' has value one would expect for δ-function approx with $\psi_s \sim 2\psi_B + 3kT/q$