Homework #7 Solutions

1. $$\mu_{eff} = \frac{\mu_0}{1 + \alpha \bar{\mathcal{E}}_{vert}\,\mu_0}$$

$$|\bar{\mathcal{E}}_{vert}| = \frac{1}{2}\left[\frac{|Q_S'|}{K_S\epsilon_0} + \frac{|Q_B'|}{K_S\epsilon_0}\right] = \frac{1}{2K_S\epsilon_0}\left(2|Q_B'| + |Q_I'|\right)$$

$$Q_I' = C_{ox}'\left(V_{GC} - V_T'\right), \quad Q_B' = \sqrt{2qK_S\epsilon_0 N_a\left(2\psi_B + V_{CS} + V_{SB}\right)}$$

V_{CB}

$$|\bar{\mathcal{E}}_{vert}| = \sqrt{\frac{2qN_a}{K_S\epsilon_0}}\left(2\psi_B + V_{CS} + V_{SB}\right)^{1/2} + \frac{C_{ox}'}{2K_S\epsilon_0}\left(V_{GC} - V_T'\right)$$

$$V_T' = V_T + \delta V_{CS} = V_T + (m-1)V_{CS} \qquad V_T' = V_t + mV$$

$$V_{GC} - V_T' = V_{GS} - V_T - V_{CS} - (m-1)V_{CS}$$

$$= V_{GS} - V_T - mV_{CS}$$

$$|\bar{\mathcal{E}}_{vert}| = \sqrt{\frac{2qN_a}{K_S\epsilon_0}}\left(2\psi_B + V_{CS} + V_{SB}\right)^{1/2} + \frac{C_{ox}'}{2K_S\epsilon_0}\left(V_{GS} - V_T - mV_{CS}\right)$$

Several mobility models can give this field dependence (most are more complicated). In general, they use the local $\mathcal{E}_\perp$ (field perpendicular to direction of current flow), which averaged over inversion charge gives an average value similar to $|\bar{\mathcal{E}}_{vert}|$ above.

With Matthiessen's rule $\left(\frac{1}{\mu} = \sum_i \frac{1}{\mu_i}\right)$, a term of the form $\frac{1}{\alpha \mathcal{E}_\perp}$ along w/ μ_0 gives $\mu = \left(\alpha\mathcal{E}_\perp + \frac{1}{\mu_0}\right)^{-1} = \mu_0/(1+\alpha\mathcal{E}_\perp\mu_0)$. This form is possible with Lombardi Model using just 1st term of Eq. 169, or with University of Bologna model with $\lambda = 1$ in Eq. 177.

3

$$\frac{I_{ds}^{sub}}{W} = \mu_{eff} C_{ox}' \frac{1}{L} (m-1)\left(\frac{kT}{q}\right)^2 \exp\left(-\frac{qV_T}{mkT}\right) < 10^{-9}\frac{A}{\mu m} \qquad V_{gs}=0$$

$$I_{ds}^{on} = I_{ds}^{sat} = \mu_{eff} C_{ox}' \frac{W}{L} \left(\frac{(V_{gs}-V_T)^2}{2m}\right) \qquad w/\ V_{gs} = 2V = V_{dd}$$

$$V_T = \Phi_{ms} + 2\psi_B + \frac{qN_a W_{dm}}{C_{ox}'} + \frac{qN'}{C_{ox}'} - 24\frac{t_{ox}}{W_{dm}}\sqrt{\psi_{bi}(\psi_{bi}+V_{ds})}\, e^{-\frac{\pi L}{2mW_{dm}}}$$

$$W_{dm} = \sqrt{\frac{4K_s\epsilon_0\psi_B}{qN_a}}, \quad m = 1 + \frac{3t_{ox}}{W_{dm}}$$

As rule of thumb: $W_{dm} > 6t_{ox} = 0.024\,\mu m = 24\,nm \Rightarrow N_a < 2.2\times10^{18}\,cm^{-3}$

$L > 2(W_{dm} + 3t_{ox}) = 0.072\,\mu m = 72\,nm$

Start with $N_a = \alpha * 10^{18}\,cm^{-3}$, $L = 0.1\,\mu m + 0.001\,\mu m * \beta = (100+\beta)\,nm$

~~Wdm~~ $\psi_B \cong 0.47\,V$, $\phi_{ms} \cong -1.05\,V$, $C_{ox}' = 8.6\times10^{-7}\frac{F}{cm^2}$

$V_T = -0.11\,V + 0.65\alpha^{1/2} + 0.19\gamma - 4.88\alpha^{1/2} e^{-\pi(100+\beta)/2(\alpha^{-1/2}+0.34)35}$

$W_{dm} = 3.5\times10^{-6}\,cm\,(\alpha)^{-1/2} \cong 35\,\alpha^{-1/2}\,nm$

$N' = 10^{12}\gamma\ cm^{-2}$

$= -0.11 + 0.65\alpha^{1/2} + 0.19\gamma - 0.17\alpha^{1/2} e^{-3\beta/100}$

~~$m = 1 + 0.34\alpha^{1/2}$~~ $\cong 1.34$

~~$\alpha \cong 1+\epsilon$~~

$e^{-\pi(100+\beta)/70(1.34)} \cong e^{-\pi\left(\frac{100+\beta}{70(1.34)}\right)} = 0.035 * e^{-3\beta}$

$\mathcal{E}_{eff} \cong \frac{|Q_d|}{K_s\epsilon_0}$, $\mathcal{E}_{eff}^{on} \cong \frac{|Q_d| + \frac{1}{2}C_{ox}'(V_{gs}-V_T)}{K_s\epsilon_0} = 0.54\alpha^{1/2}\frac{MV}{cm} + 0.66\frac{MV}{cm}$

~~$\cong \frac{qW_{dm}N_a}{K_s\epsilon_0} = 5.4\alpha^{1/2}\times10^{5}\frac{V}{cm} \Rightarrow \mu_{eff} \cong 100\frac{cm^2}{V s}$~~ $\mu_{eff}^{on} \cong 180\frac{cm^2}{V s}$

From Fig 3.13

For off current, worst case is $N_a^{wc} = N_a(0.95)$

$~~N'_{wc} = N'(0.95)~~$

$L^{wc} = L - 20\,nm$

$$10^{-9}\frac{A}{\mu m} > \frac{I_{ds}^{sub}}{W} = 100\frac{cm^2}{V \cdot s}\left(8.6\times10^{-7}\frac{F}{cm^2}\right)\frac{1}{(0.1 + \beta/1000)}\left(0.34\,\alpha^{1/2}\right)\left(\frac{kT}{q}\right)^2$$

$$* \exp\left(-\frac{q V_T}{(1+0.34\alpha^{1/2})\,kT}\right)$$

Assume $\alpha = 0.95$ (worstcase effect) and $\beta \cong -20$

$V_T^{wc} \gtrsim 0.19\,V$ $\quad V_T^{nom}(\alpha=1, \beta=0) \cong 0.37 + 0.19\,\gamma$

$V_T^{wc}(\alpha = 0.95, \beta = -20) = 0.22 + 0.19\,\gamma$

For $\gamma = 0$, could increase $|\beta|$ to 22 and have $V_T^{wc} = 0.19V$

This corresponds to $L_{nom} = 97\,nm$

For on current, worst-case is $N_a^{wc} = 1.05\, N_a$

$$N'_{wc} = 1.05\, N'$$

$$L^{wc} = L + 20\,\mathrm{nm}$$

$$\frac{I_{ds}^{on}}{W} = \frac{I_{ds}^{sat}}{W} = \mu_{eff} C'_{ox} \frac{1}{L} \frac{(V_{gs} - V_T^{wc})^2}{2m} = 180 \frac{\mathrm{cm}^2}{\mathrm{V{\cdot}s}} \left(8.6\times10^{-7} \frac{\mathrm{F}}{\mathrm{cm}^2}\right) \frac{1}{0.12\,\mu\mathrm{m}} \frac{(2-0.46)^2}{2(1.34)}$$

$$= 11.4\ \mathrm{A/cm} = 1.14\times10^{-3}\ \mathrm{A}/\mu\mathrm{m}$$

$$V_T^{wc} = -0.11 + 0.65(1.05)^{1/2} - 0.17(1.05)^{1/2} e^{-60/100} = 0.46\,\mathrm{V}$$

$$\tau = \frac{(4 L C'_{ox} + 0.5\,\mu\mathrm{m}\, C'_j)\, 2\,\mathrm{V}}{I_{ds}^{on}/W} \qquad C_j = 4\times10^{-7} \frac{\mathrm{F}}{\mathrm{cm}^2}$$

$$= \frac{\left[4 (0.12\times10^{-4}\,\mathrm{cm})\left(8.6\times10^{-7} \frac{\mathrm{F}}{\mathrm{cm}^2}\right) + 5\times10^{-5}\,\mathrm{cm}\left(4\times10^{-7}\frac{\mathrm{F}}{\mathrm{cm}^2}\right)\right] 2\,\mathrm{V}}{11.4\ \mathrm{A/cm}} = 1.1\times10^{-14}\,\mathrm{s}$$

$$= 11\,\mathrm{fs}$$

W/ v_{sat} $\quad \dfrac{\mu_{eff} V_{ds}}{v_{sat} L} = \dfrac{\left(180 \frac{\mathrm{cm}^2}{\mathrm{V{\cdot}s}}\right)(2\,\mathrm{V})}{\left(8\times10^6 \frac{\mathrm{cm}}{\mathrm{s}}\right)(0.12\,\mu\mathrm{m})} = 3.75$ so

velocity saturation is significant

Using 3.78 instead,

$$\frac{I_{dsat}}{W} = C'_{ox} v_{sat} (V_{gs} - V_T) \frac{\sqrt{1+\xi} - 1}{\sqrt{1+\xi} + 1} \qquad \xi = 2.15$$

$$= 2.96\ \mathrm{A/cm}, \quad \tau = 4.2\times10^{-14}\,\mathrm{s} = 42\,\mathrm{fs}$$

To optimize, check whether lower (or higher) doping would give better results (e.g., try $8\times10^{17}\,\mathrm{cm}^{-3}$ or $1.2\times10^{18}\,\mathrm{cm}^{-3}$) and repeat.