EE 531

# Homework #1 Solutions

1. $$U = \frac{pn - n_i^2}{\tau_p\left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

$$\tau_p = (\sigma_p N_t v_{thp})^{-1} = \begin{cases} 1.67\times10^{-5}\text{ s} & \text{for deep levels} \\ 1.67\times10^{-11}\text{ s} & \text{for shallow levels}\end{cases}$$

$$v_{thp} = 6\times10^6 \frac{\text{cm}}{\text{s}}$$

$$\tau_n = (\sigma_p N_t v_{thn})^{-1} = \begin{cases} 10^{-5}\text{ s} & \text{for deep levels} \\ 10^{-11}\text{ s} & \text{for shallow levels}\end{cases}$$

Consider recombination to be sum of recombination through deep and shallow levels

$$U = U_s + U_d$$

$$E_i = E_V + \frac{E_g}{2} + \frac{kT}{2}\ln\frac{N_V}{N_C} = E_V + 0.55\text{eV} \qquad (E_g = 1.12\text{eV})$$

(a) $$U_s = \frac{pn - n_i^2}{\tau_{ps}\left[n + n_i \exp\left(\frac{1.07 - 0.55\text{eV}}{kT}\right)\right] + \tau_{ns}\left[\underset{p \ll n}{\cancel{p}} + \underset{\text{small}}{\cancel{n_i \exp\left(\frac{0.55 - 1.07\text{eV}}{kT}\right)}}\right]}$$

$$\cong \frac{\Delta p\, n_0}{\tau_{ps}\left[n_0 + n_i \exp\left(\frac{1.07 - 0.55\text{eV}}{kT}\right)\right)} = \frac{\Delta p}{\tau_s}$$

$$\tau_s = 1.67\times10^{-11}\text{ s}\left[1 + \frac{1.5\times10^{10}}{5\times10^{18}}\exp\left(\frac{0.52}{0.0259}\right)\right] = 4.3\times10^{-11}\text{ s}$$

$$U_d \cong \frac{pn - n_i^2}{\tau_{pd}\, n_0} \cong \frac{\Delta p}{\tau_{pd}} \qquad \tau = \frac{\tau_d \tau_s}{\tau_d + \tau_s} \cong \tau_s = 4.3\times10^{-11}\text{s}$$

Dominated by shallow defects $\tau_s \ll \tau_d = \tau_{pd}$

(b) From (a) $\tau_s \cong \tau_{ps} \left[ 1 + \frac{n_1}{n_0} \right]$ $n_1 = 7.9 \times 10^{18}$

In the range of interest $(10^{17} - 3 \times 10^{18}\ cm^{-3})$,

$n_1 >> n_0$, so $\tau_s \cong \tau_{ps} \left( \frac{n_1}{n_0} \right)$ ~~to~~

If $\tau_{ps}$ = constant (not a function of $n_0$), then

$\tau_s \propto (n_0)^{-1}$ as ~~for~~ line on plot

$\tau_s(10^{17}) \cong 2.5 \times 10^{-5} = \tau_{ps} \left( \frac{7.9 \times 10^{18}}{10^{17}} \right) = 79 \tau_{ps}$

$\tau_{ps} = 3.2 \times 10^{-7}\ s$ For $N_{ts} = 10^{18}\ cm^{-3}$, $\sigma_{ps} = \frac{1}{3.2 \times 10^{-7} (10^{18})\, 6 \times 10^{6}}$

$= 5.2 \times 10^{-19}$

We would actually expect $N_{ts}$ to increase w/ $n_0$

Assuming $N_{ts} = n_0$, $\tau_s \cong \frac{1}{\sigma_{ps} v_{thp} n_0} \left[ 1 + \frac{n_1}{n_0} \right]$

$\cong \frac{n_1}{\sigma_{ps} v_{thp} n_0^2} \quad (\propto n_0^{-2})$

Note that this is actually closer to data $(n_d^{-2})$

$\tau_{\cancel{s}}(10^{18}) = \tau_s(10^{18}) \cong 10^{-6}\ s = \frac{7.9 \times 10^{18}\ cm^{-3}}{\sigma_{ps} (6 \times 10^{6} \frac{cm}{s}) (10^{18}\ cm^{-3})^2}$

$\sigma_{ps} = 1.3 \times 10^{-18}\ cm^2$, $\tau_{ps} = 1.3 \times 10^{-7}\ s \left( \frac{10^{18}\ cm^{-3}}{n_0} \right)$

Note that for $n_0 > 3 \times 10^{18}\ cm^{-3}$ $n_1 \sim n_0$, so $\tau \sim n_0^{-1}$

(c) For $n_0 \sim 10^{19} - 10^{20}$ Auger dominates

$$\tau_p = \frac{1}{K_n n_0^2} \quad (l.l.i.) \qquad \tau_p(3\times10^{19}) \cong 10^{-8}\,s$$

Thus $K_n = \frac{1}{\tau_p n_0^2} = \frac{1}{(10^{-8}s)(3\times10^{19}cm^{-3})^2} = 1.1\times10^{-31}\,cm^{+6}s^{-1}$

(d) For $10^{19}\,cm^{-3}$, dominated by Auger recombination

$$U_{Auger} = K_n n^2 p + K_p p^2 n \cong K_n (n_0+\Delta p)^2 \Delta p + K_p (\Delta p)^2 (n_0+\Delta p)$$

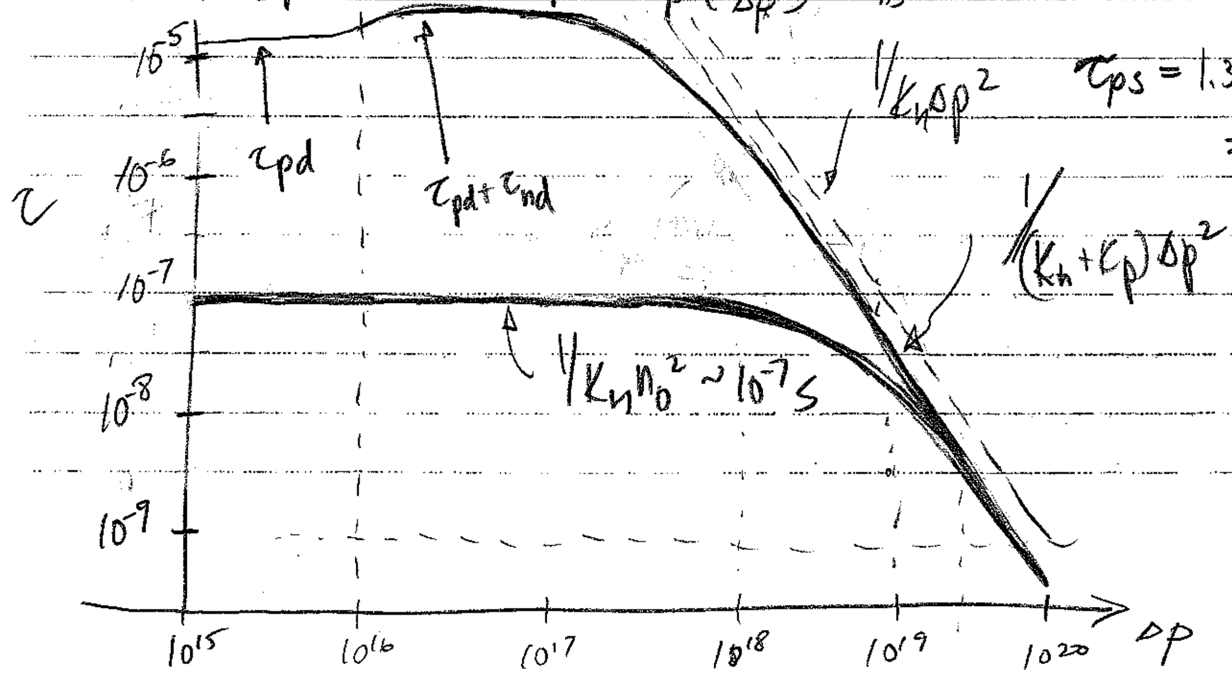
$$\tau = \frac{\Delta p}{U} = \frac{1}{K_n(n_0+\Delta p)^2 + K_p(\Delta p)(n_0+\Delta p)} \cong \begin{cases} \frac{1}{K_n n_0^2} \text{ for } \Delta p \ll n_0 = 10^{19} \\ \frac{1}{(K_n+K_p)\Delta p^2} \text{ for } \Delta p \gg n_0 \end{cases}$$

For $n_0 = 10^{16}$, $U = U_{shallow} + U_{deep} + U_{Auger}$

For l.l.i. $(\Delta p \ll 10^{16})$, $\tau \cong \tau_{pd} = 1.67\times10^{-5}s$

For $10^{16} \ll \Delta p \ll 10^{19}$ (high level injection), $U_d \cong \frac{\Delta p^2}{\tau_{pd}(\Delta p) + \tau_{nd}(\Delta p)} \cong \frac{\Delta p}{\tau_{pd}+\tau_{nd}} = \frac{\Delta p}{2.67\times10^{-5}s}$

$$U_s \cong \frac{\Delta p^2}{\tau_{ps}(\Delta p + n_1) + \tau_{ns}\Delta p} \cong \frac{\Delta p}{\tau_{ps}\left(\frac{n_1}{\Delta p}\right) + \tau_{ns}}$$

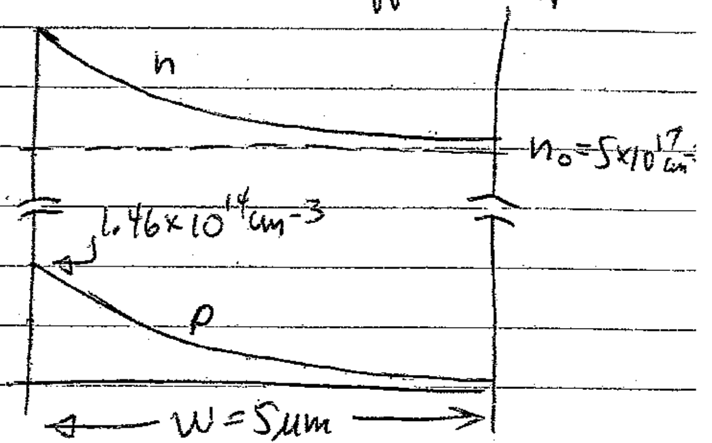


$$\tau_{ps} = 1.3\times10^{-7}\left(\frac{10^{18}}{10^{10}}\right) = 1.3\times10^{-5}\,s \sim \tau_{pd}$$

so shallow traps never dominate

2 (a) $L_p = \sqrt{D_p \tau_p} = 6.3 \mu m \sim W = 5 \mu m$

Assume low-level injection, charge neutrality, diffusion approximation



$n = n_0 + \Delta n \cong n_0$

$p = p_0 + \Delta p \cong \Delta p$

$$\frac{d\Delta p}{dt} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p}$$

$\underset{\text{in s.s}}{0} =$

$$\Delta p = Ae^{-x/L_p} + Be^{x/L_p}$$

$\underline{\underline{BCs}}$ $\underline{\underline{x=0}}$ $G_{Ls} = J_p(0)/q$ $\underline{\underline{x=W}}$ $\frac{1}{q} J_p(W) = U_s$

$$10^{18} cm^{-2} s^{-1} = -D_p \frac{d\Delta p}{dx}\Big|_{x=0} \qquad -D_p \frac{d\Delta p}{dx}\Big|_{x=W} = S\Delta p(W)$$

$$10^{18} cm^{-2} s^{-1} = \frac{D_p}{L_p}(A - B) \qquad \frac{D_p}{SL_p}\left(Ae^{-W/L_p} - Be^{+W/L_p}\right) = \left(Ae^{-W/L_p} + Be^{W/L_p}\right)$$

$$A - B = 1.6\times10^{14} cm^{-3} \qquad 0.63\left(\frac{A}{2.2} - 2.2B\right) = \frac{A}{2.2} + 2.2B$$

$$A = -21.3B \qquad B = -7.2\times10^{12} cm^{-3}, A = 1.53\times10^{14} cm^{-3}$$

(b) $U_s = S\Delta p(W) = 10^4 \frac{cm}{s}\left[\frac{1.53}{2.2} - 0.07 * 2.2\right] * 10^{14} cm^{-3} = 5.4\times10^{17} cm^{-2} s^{-1}$

$\frac{U_s}{G_{Ls}} = 0.54 \Rightarrow$ 54% recombine at W = 5um
46% recombine in bulk

(c) $\Delta n \cong \Delta p$ , $D_n \cong 10 \, \mathrm{cm^2/s}$ (from graph or $2.5 * D_p$)

$$J_n^{diff} = q D_n \frac{d(\Delta n)}{dx} = q D_n \left[ -\frac{A}{L_p} e^{-x/L_p} + \frac{B}{L_p} e^{x/L_p} \right]$$

$$= (1.6\times10^{-19}\,\mathrm{C})\left(10\,\frac{\mathrm{cm^2}}{\mathrm{s}}\right)\left[ -\frac{1.53\times10^{14}\,\mathrm{cm^{-3}}\, e^{-x/L_p}}{6.3\times10^{-4}\,\mathrm{cm}} - \frac{7.2\times10^{12}\,\mathrm{cm^{-3}}\, e^{x/L_p}}{6.3\times10^{-4}\,\mathrm{cm}} \right]$$

$$= -\left[ 0.39\, e^{-x/6.3\mu m} + 0.02\, e^{x/6.3\mu m} \right] \mathrm{A/cm^2}$$

(d) $J_p \cong J_p^{diff} = -\frac{D_p}{D_n} J_n^{diff}$ since $\Delta n \cong \Delta p$

but $J_p = -J_n$ (same fluxes, opposite currents)

$$= -(J_n^{diff} + J_n^{drift})$$

$$J_n^{drift} = \left[ \frac{D_p}{D_n} - 1 \right] J_n^{diff} = -0.6 J_n^{diff} = -\left[ 0.23\, e^{-x/6.3\mu m} + 0.01\, e^{x/6.3} \right.$$

$$= q\mu_n n \mathcal{E} = (1.6\times10^{-19}\,\mathrm{C})\left( \frac{10\,\mathrm{cm^2/s}}{0.026\,\mathrm{V}} \right)(5\times10^{17}\,\mathrm{cm^{-3}})\, \mathcal{E}$$

$$\mathcal{E} = \frac{J_n^{drift}}{30.8\,(\Omega\text{-cm})^{-1}} = -\left[ 7.5\times10^{-3}\, e^{-x/6.3\mu m} + 3.5\times10^{-4}\, e^{x/6.3\mu m} \right]$$

(e) $J_p^{drift} = q\mu_p p \mathcal{E} = J_n^{drift}\left(\frac{\mu_p}{\mu_n}\right)\left(\frac{p}{n}\right) =$

$$= -\frac{(1.53\times10^{14}\,\mathrm{cm^{-3}}\, e^{-x/L_p} + 7.2\times10^{12}\,\mathrm{cm^{-3}}\, e^{x/L_p})}{(2.5)(5\times10^{18}\,\mathrm{cm^{-3}})}\left( 0.23\, e^{-x/L_p} + 0.01\, e^{x/L_p} \right)$$

$$\frac{J_p^{drift}}{J_p^{diff}} = \frac{J_n^{drift}\left(\cancel{\mu_p/\mu_n}\right)(p/n)}{-\cancel{(D_p/D_n)}\, J_n^{diff}} = \frac{-0.6\, \cancel{J_n^{diff}}\, (p/n)}{-\cancel{J_n^{diff}}} = 0.6\, \frac{p}{n} < 1.8\times10^{-5}$$

So Diffusion Approximation is excellent

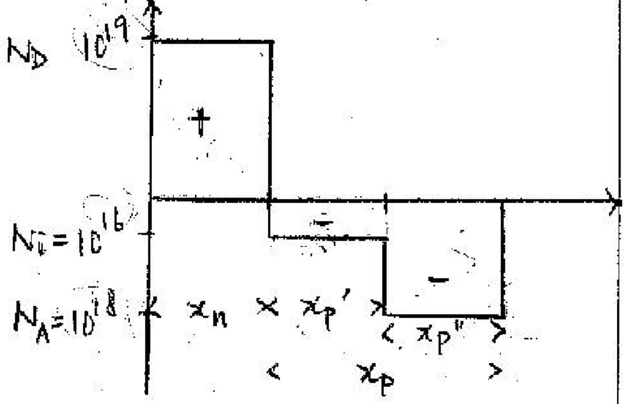
(f) $$\frac{\rho}{q} = \frac{K\epsilon_0}{q}\frac{d\mathcal{E}}{dx} = \frac{12\left(8.854\times10^{-14}\,\frac{\mathrm{F}}{\mathrm{cm}}\right)}{(1.6\times10^{-19}\,\mathrm{C})}\left[ \frac{7.5\times10^{-3}\,\mathrm{V/cm}\, e^{-x/L_p}}{6.3\times10^{-4}\,\mathrm{cm}} - \frac{3.5\times10^{-4}\,\mathrm{V/cm}}{6.3\times10^{-4}\,\mathrm{cm}} e^{x/L_p} \right]$$

$$= 7.9\times10^{10}\, e^{-x/L_p} - 3.7\times10^{9}\, e^{x/L_p} << \Delta p, \Delta n$$

Again, an excellent approx.

HW6 Sol$^{n}$

1 (a) $N_D = 10^{19}\,cm^{-3}$ (n), $N_i = 10^{16}\,cm^{-3}$ (p'), $N_A = 10^{18}\,cm^{-3}$ (P)



First, we need to find $x_n$, $x_p$ using charge neutrality,

$$N_D x_n = N_i x_p' + N_A x_p''$$

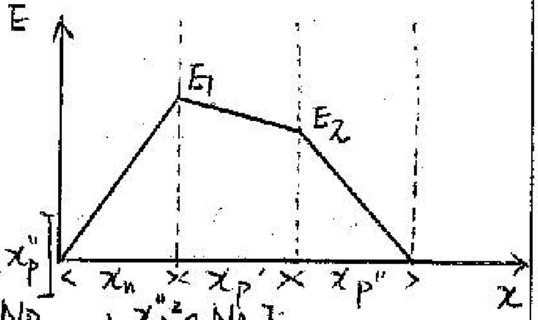
We do know & can calculate $\phi_i$ right away,

$$\phi_i = \frac{kT}{q} \ln\left(\frac{N_D N_A}{(n_i)^2}\right) \cong 0.0259\,meV \ln\left(\frac{(10^{18})(10^{19})}{(10^{10})^2}\right) = \underline{\underline{1.01V}}$$

$$E_1 = \frac{q N_D x_n}{K\epsilon_0}, \quad E_2 = \frac{q N_A x_p''}{K\epsilon_0}$$

$$= E_1 - \frac{q N_i x_p'}{K\epsilon_0}$$

dielectric constant



$$\phi_i = -\int E dx = -\left[\frac{1}{2} x_n E_1 + \frac{1}{2}(E_1 + E_2) x_p' + \frac{1}{2} E_2 x_p''\right]$$

$$= -\left[\frac{1}{2}\frac{x_n^2 q N_D}{K\epsilon_0} + \frac{1}{2}\frac{x_n x_p' q N_D}{K\epsilon_0} + \frac{1}{2}\frac{x_p'' x_p' q N_D}{K\epsilon_0} + \frac{1}{2}\frac{x_p''^2 q N_A}{K\epsilon_0}\right]$$

$$\frac{2K\epsilon_0 \phi_i}{q} = x_n^2 N_D + x_n x_p' N_D + x_p x_p' N_A + x_p'^2 N_A.$$

$\Downarrow$ a few algebra steps

$$13.01 = 110\left(\frac{x_p''}{x_p'}\right)^2 + 200.2\left(\frac{x_p''}{x_p'}\right) + 1.001$$

$$\frac{x_p''}{x_p'} = 0.058 \quad \text{(negative result was disgarded)}$$

$x_p' = 0.1\,\mu m$ (given) → $x_p'' = 0.1 \times 0.058\,\mu m = \underline{\underline{5.8\,nm}}$

$$x_n = \frac{N_i x_p' + N_A x_p''}{N_D} = \frac{(0.1\mu m)(10^{16}cm^{-3}) + (5.8nm)(10^{18}cm^{-3})}{(10^{19}cm^{-3})}$$

$$= \underline{\underline{0.68\,nm}}$$

1(b) Compare $\mathcal{E}_{max}$.

$$\text{Emax for pin diode} = E_1 = \frac{(1.6\times10^{-19}C)(10^{19}cm^{-3})(\overset{x_n}{\downarrow}6.8\times10^{-8}cm)}{(11.7)(8.85\times10^{-14}F/cm)}$$

$$= \underline{\underline{1.05\times10^{5}\,Vcm^{-1}}}$$

$$\text{Emax for pn diode} = \left[\frac{2q\,\phi_i N_A N_D}{K\epsilon_0(N_A+N_D)}\right]^{\frac{1}{2}} = \left[\frac{2q(1.01V)(10^{19}cm^{-3})(10^{18}cm^{-3})}{(11.7)(8.85\times10^{-14}F/cm)(1.1\times10^{19}cm^{-3})}\right]^{\frac{1}{2}}$$

$$= \underline{\underline{5.33\times10^{5}\,Vcm^{-1}}} \gg E_1$$

1(c) pin, from 1(a), we see that $x_p' \gg x_n$ or $x_p''$, which means that the depletion width $x_d \simeq x_p'$.

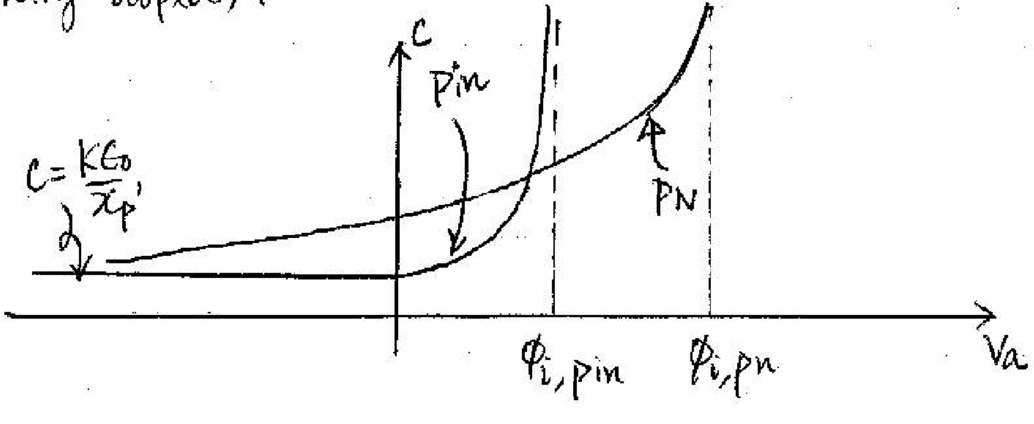
Under reverse bias, the intrinsic region is fully depleted, so,

$$C \simeq \frac{K\epsilon_0}{x_p'} \simeq \text{constant.}$$

As bias voltage becomes increasingly positive (forward bias), the intrinsic region would not be fully depleted at some point.

$$C = \left[\frac{q K\epsilon_0}{2\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(\phi_i - V_A)}\right]^{\frac{1}{2}} \quad \text{as } V_A \to \phi_i \Rightarrow C \to \infty.$$

The device becomes a one-sided p-n juction (p-side very lightly doped).



$\phi_{i,pn} > \phi_{i,pin}$

3) $N_a = 5\times10^{18}\ cm^{-3}$ $\quad N_d = 2\times10^{17} cm^{-3}$

$\tau_n = \tau_p = 0.02\mu s$ (P-region) $\quad W_p = 100nm = 100\times10^{-7}\ cm$

$\tau_n = \tau_p = 0.5\mu s$ (n-region) $\quad W_n = 700\mu m = 700\times10^{-4}\ cm$

a) No recombination in the depletion region

In order to sketch this, we will need information for xn, xp, the current densities at the edges of the depletion region, and at the contacts.

It's also useful to check whether this is a long or short diode.

$$N_a = 5\times10^{18} cm^{-3} \Longrightarrow D_n = \left(\frac{KT}{q}\right)\left(175 \frac{cm^2}{Vs}\right) = 4.38\ cm^2/s$$

$$N_d = 2\times10^{17} cm^{-3} \Longrightarrow D_p = \left(\frac{KT}{q}\right)\left(200 cm^2/Vs\right) = 5\ cm^2/s$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{4.38\times0.02\times10^{-6}}\ cm = 2.96\times10^{-4}\ cm >> W_p$$

(Short diode at P side)

$$L_p = \sqrt{D_p \tau_p} = \sqrt{5\times0.5\times10^{-6}}\ cm = 1.58\times10^{-3}\ cm$$

To get xn and xp, we need $\varphi_i$ :

$$\varphi_i = \frac{KT}{q}\ln\left(\frac{N_d N_a}{n_i^2}\right) = 0.921\ V$$

$\Longrightarrow$

3 a)

$$x_n = \sqrt{\frac{2K\epsilon_0(\varphi_i - V_A)}{q} \frac{N_a}{N_d} \left(\frac{1}{N_a + N_d}\right)} = 3.70 \times 10^{-6} \text{ cm}$$

$$x_p = \sqrt{\frac{2K\epsilon_0(\varphi_i - V_A)}{q} \frac{N_d}{N_a} \left(\frac{1}{N_d + N_a}\right)} = 1.48 \times 10^{-7} \text{ cm}$$

i) $J_p(x_n) = q \dfrac{D_p p_{no}}{L_p}\left(\exp\left(\dfrac{qV_a}{kT}\right) - 1\right)$

$= (1.6 \times 10^{-19} \text{ c}) \dfrac{5 \text{ cm}^2/\text{s} \times 500 \text{ cm}^{-3}}{1.58 \times 10^{-3} \text{ cm}} \left(e^{\left(\frac{0.7}{0.025}\right)} - 1\right)$

$= \underline{\underline{0.366 \text{ Acm}^{-2}}}$

ii) $J_p(x_p) = J_p(x_n) = 0.366 \text{ Acm}^{-2}$ (No recombination)

iii) $J_n(-W_p) = q \dfrac{D_n n_{po}}{W_p}\left[\exp\left(\dfrac{qV_a}{kT}\right) - 1\right] = J_n(-x_p)$

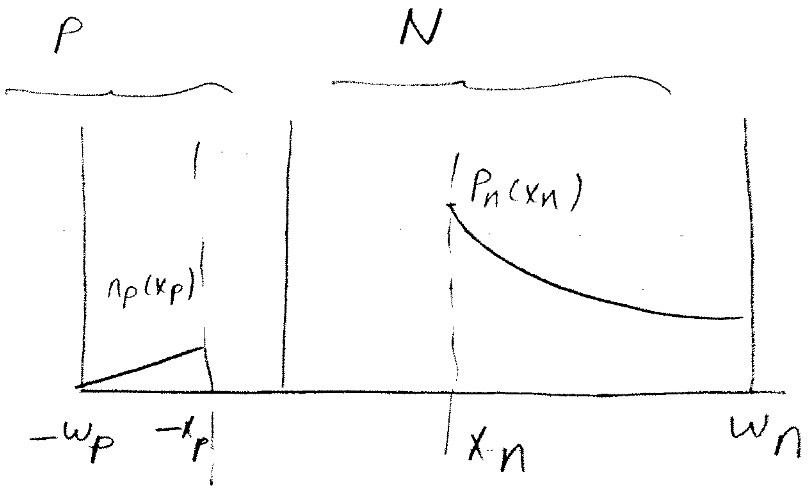
$= (1.6 \times 10^{-19} \text{ c}) \dfrac{4.38 \times 20 \text{ cm}^{-1} \text{s}^{-1}}{100 \times 10^{-7} \text{ cm}} \left[\exp\left(\dfrac{0.7}{0.025}\right) - 1\right]$

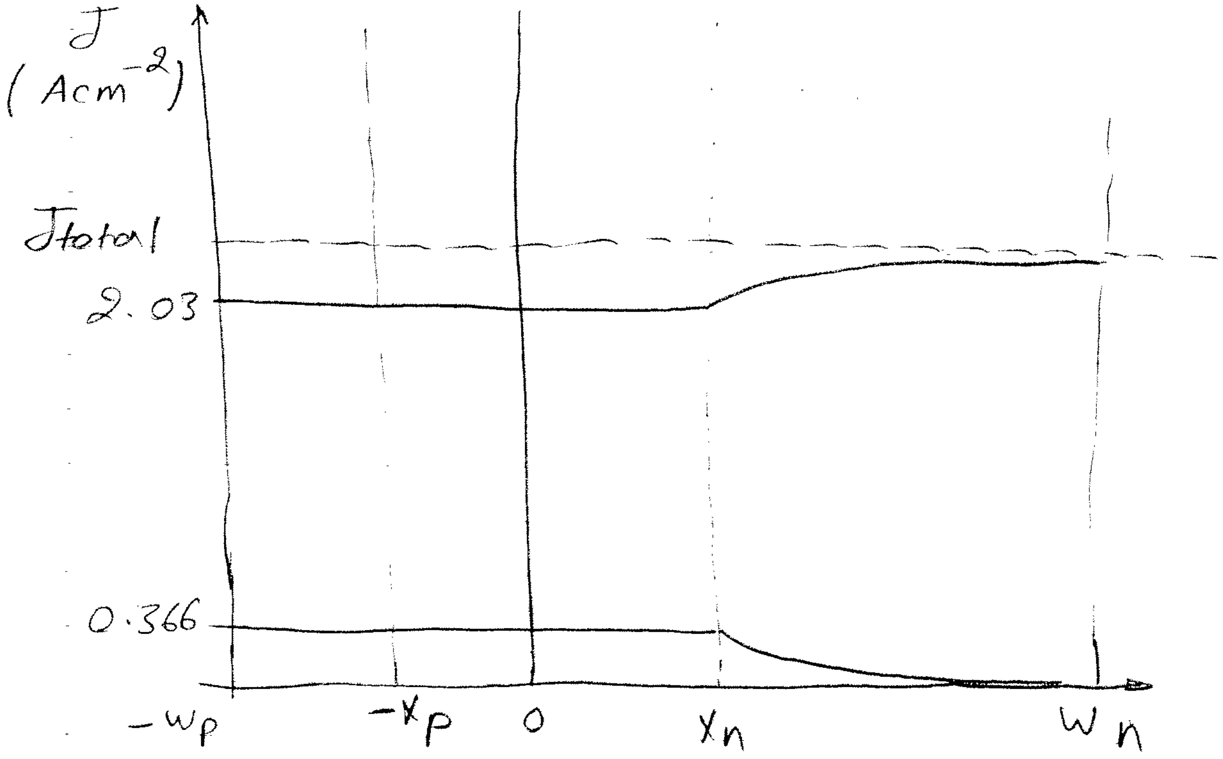
$= \underline{\underline{2.03 \text{ A cm}^{-2}}}$

iv) $J_n(W_n) \simeq J_{total} = J_p(x_n) + J_n(-x_p)$

$= 2.4 \text{ Acm}^{-2}$

3 a)





3 b) $V_A = 0.3V$ and we have recombination in the depletion region. $x'_d = x_d/10 = \frac{6.45}{10} \times 10^{-6}$ cm

$= \underline{0.645 \times 10^{-6} \text{ cm}}$

(i) $J_p(x_n) = q\frac{D_p}{L_p} p_{no}\left[e^{\left(\frac{qV_a}{0.025}\right)} - 1\right] = \underline{4.1 \times 10^{-8} \text{ Acm}^{-2}}$

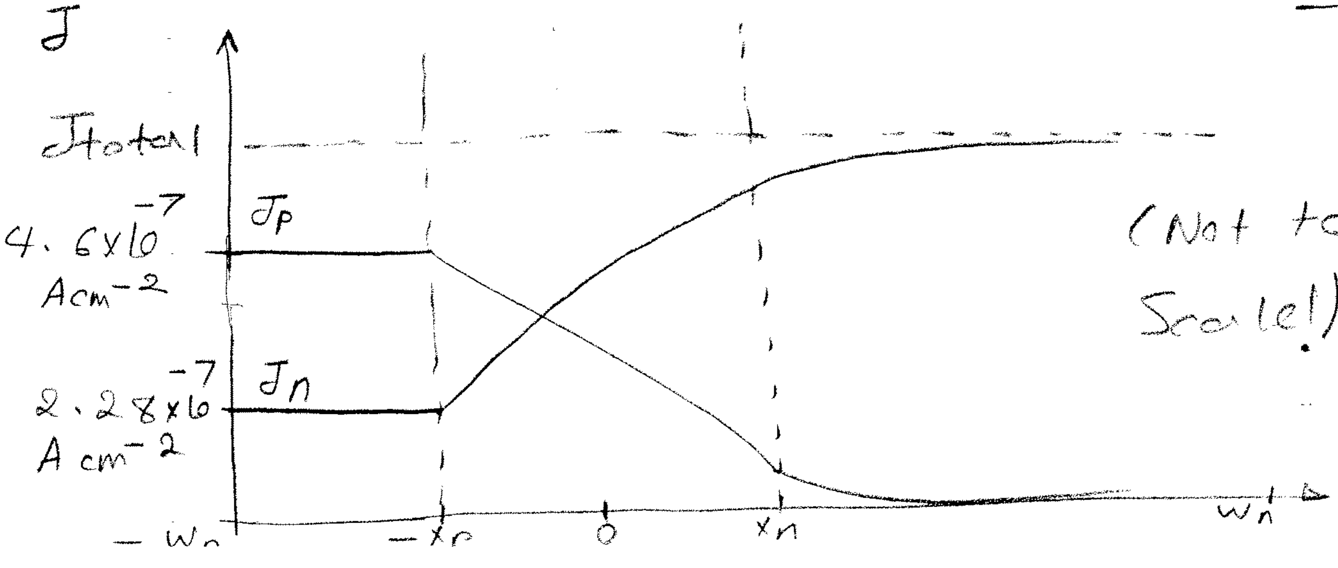
(ii) Need to calculate recombination

$$J_{rec} = \underbrace{\frac{q x_d' n_i}{2\tau_o}}_{\tau_n + \tau_p \rightarrow \text{ using n-region values.}} \exp\left(\frac{qV_a}{2kT}\right) = \underline{4.2 \times 10^{-7}} \text{ Acm}^{-2}$$

$\Rightarrow J_p(-x_p) = J_p(x_n) + J_{rec} \simeq \underline{4.6 \times 10^{-7}} \text{ Acm}^{-2}$

iii) $J_n(-W_p) = q\frac{D_n n_{po}}{W_p}\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] = 2.28 \times 10^{-7} \text{ Acm}^{-2}$

iv) $J_n(W_n) \simeq J_{total} = J_n(w_p) + J_p(-x_p) = \underline{6.9 \times 10^{-7} \text{ Acm}^{-2}}$



(Not to Scale!)