Homework #4 Solutions

$V = (V_{CS} + V_{SB})$

1 (a) $$Q'_{dm} = -\sqrt{2K_s \epsilon_0 q N_a (2\psi_B + V_{CB})}$$ $V_{SB} = 0$, so $V_{CB} = V_{CS}$

$$\cong -Q'_{dm}(V_{CB}=0) + \left.\frac{d Q'_{dm}}{d V_{CS}}\right|_{V_{CS}=0} V_{CS}$$

$$= -\sqrt{2K_s \epsilon_0 q N_a (2\psi_B)} + \frac{1}{2}\sqrt{2K_s \epsilon_0 q N_a}\,(2\psi_B)^{-1/2}\, V_{CS}$$

$$= -\xi\sqrt{2\psi_B} - \frac{\xi V_{CS}}{2\sqrt{2\psi_B}}$$

$$Q_I = -C'_{ox}\left[V_{GB} - \left(V_{FB} + 2\psi_B + V_{CS} + \xi\frac{\sqrt{2\psi_B}}{C'_{ox}} + \frac{\xi V_{CS}}{2C'_{ox}\sqrt{2\psi_B}}\right)\right]$$

$$= -C'_{ox}\left[V_{GS} - V_T - V_{CS} - \frac{\xi V_{CS}}{2C'_{ox}\sqrt{2\psi_B}}\right]$$

V_{CS}

$$W\,dV = \frac{I_{DS}\,dy}{-Q_I \mu'_n} \Rightarrow \int_0^L I_{DS}\,dy = \int_0^{V_{DS}} -Q_I W \mu'_n\, dV$$

$$\frac{1}{\cancel{L}}\cancel{L}\, I_{DS} = \mu'_n \frac{W}{L} C'_{ox}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\left(1 + \frac{\xi}{2C'_{ox}\sqrt{2\psi_B}}\right)\right]$$

$$m - 1 = \delta = \frac{\xi}{2C'_{ox}\sqrt{2\psi_B}} = \sqrt{\frac{\cancel{2}K_s \epsilon_0 q N_a}{\cancel{2}\psi_B}}\,\frac{1}{2C'_{ox}} = \frac{1}{C'_{ox}}\sqrt{\frac{K_s \epsilon_0 q N_a}{4\psi_B}}$$

$\frac{\epsilon_{ox}}{t}$ as in Eq. 3.22

$C'_{ox} = 3.45\times10^{-7}\ F/cm^2$, $\times 1000\times10^{-4}$, $\xi = 3.17\times10^{-7}\ \frac{F V^{1/2}}{cm^2}$, $2\psi_B = 0.87V$

$\delta = 0.49$ (rather large for a modern device)

$$V_{fb} = \phi_{ms} - \frac{Q'_{ox}}{C'_{ox}} = -0.9V - \frac{3.2\times10^{-9}}{3.45\times10^{-7}} = -0.91V$$

S_{GD} V_{GD} V_{GS}

(b) For $V_{GS} = V_{DS} = 3\,V$, transistor is in saturation

$V_{GD} < V_{T} = 1.38$, 0

In saturation:

$$I_{DS} = \mu_{eff} C_{ox}' \frac{W}{L} \frac{(V_{gs} - V_T)^2}{2m} \quad \text{(linearized model)}$$

$$I_{DS} = \mu_{eff} C_{ox}' \frac{W}{L}\left[\left(V_{gs} - V_{fb} - 2\psi_B - \frac{V_{ds}^{sat}}{2}\right) V_{ds}^{sat} - \frac{2\gamma}{3C_{ox}'}\left[(2\psi_B + V_{sb} + V_{ds}^{sat})^{3/2} - (2\psi_B + V_{sb})^{3/2}\right]\right]$$

where V_{ds}^{sat} can be found by setting $Q_i = -C_{ox}(V_{gs} - V_{fb} - 2\psi_B - V_{ds}^{sat}) + \sqrt{2K_s\epsilon_0 q N_a (2\psi_B + V_{sb} + V_{ds}^{sat})} = 0$

$$\gamma^2 (2\psi_B + V_{sb} + V_{ds}^{sat}) = C_{ox}'^2 (V_{gs} - V_{fb} - 2\psi_B - V_{ds}^{sat})^2$$

$$\cancel{V_{ds}^{sat} = V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2} - \sqrt{\left(V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2}\right)^2 + (V_{gs} - V_{fb} - 2\psi_B)^2 + \frac{\gamma^2}{C_{ox}'^2}(2\psi_B + V_{sb})}}$$

$$V_{ds}^{sat} = V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2} - \sqrt{\frac{\gamma^2}{2C_{ox}'^2}\left(2(V_{gs} - V_{fb} + V_{sb}) + \frac{\gamma^2}{2C_{ox}'^2}\right)}$$

$$= 1.19\,V$$

$$m = \frac{(V_{gs} - V_T)^2}{2\left[\left(V_{gs} - V_{fb} - 2\psi_B - \frac{V_{ds}^{sat}}{2}\right) V_{ds}^{sat} - \frac{2\gamma}{3C_{ox}'}\left((2\psi_B + V_{sb} + V_{ds}^{sat})^{3/2} - (2\psi_B + V_{sb})^{3/2}\right)\right]}$$

$$V_T = V_{fb} + 2\psi_B + \frac{\gamma}{C_{ox}'}(2\psi_B + V_{sb})^{1/2} = 1.38\,V$$

$$m = \frac{(3 - 1.38)^2}{2\left[\left(3 + 0.91 - 0.87 - \frac{1.19}{2}\right)1.19 - \frac{2}{3}\left(\frac{3.17}{3.45}\right)\left[(0.87 + 2 + 1.19)^{3/2} - (0.87 + 2)^{3/2}\right]\right]}$$

$$= 1.50 \quad (\delta = 0.5)$$

$V_{GD}=3-1.5=1.5 > V_T$ non sat

For $V_{DS}=1.5V$, still in saturation, so $\delta=0.5$ gives correct current

the error due to $\delta=0.49$ is a factor of $\frac{m_1}{m_2}=\frac{1.5}{1.49}\approx 1.01$

or less than 1%.

Note that the agreement is particularly good for this example since ψ_s at source is particularly large due to body bias ~~and~~ (2.87V) and doesn't increase that much at pinch-off due to relatively high V_T. Thus the dependence of charge in depletion region on V_{GS} is nearly linear as assumed.